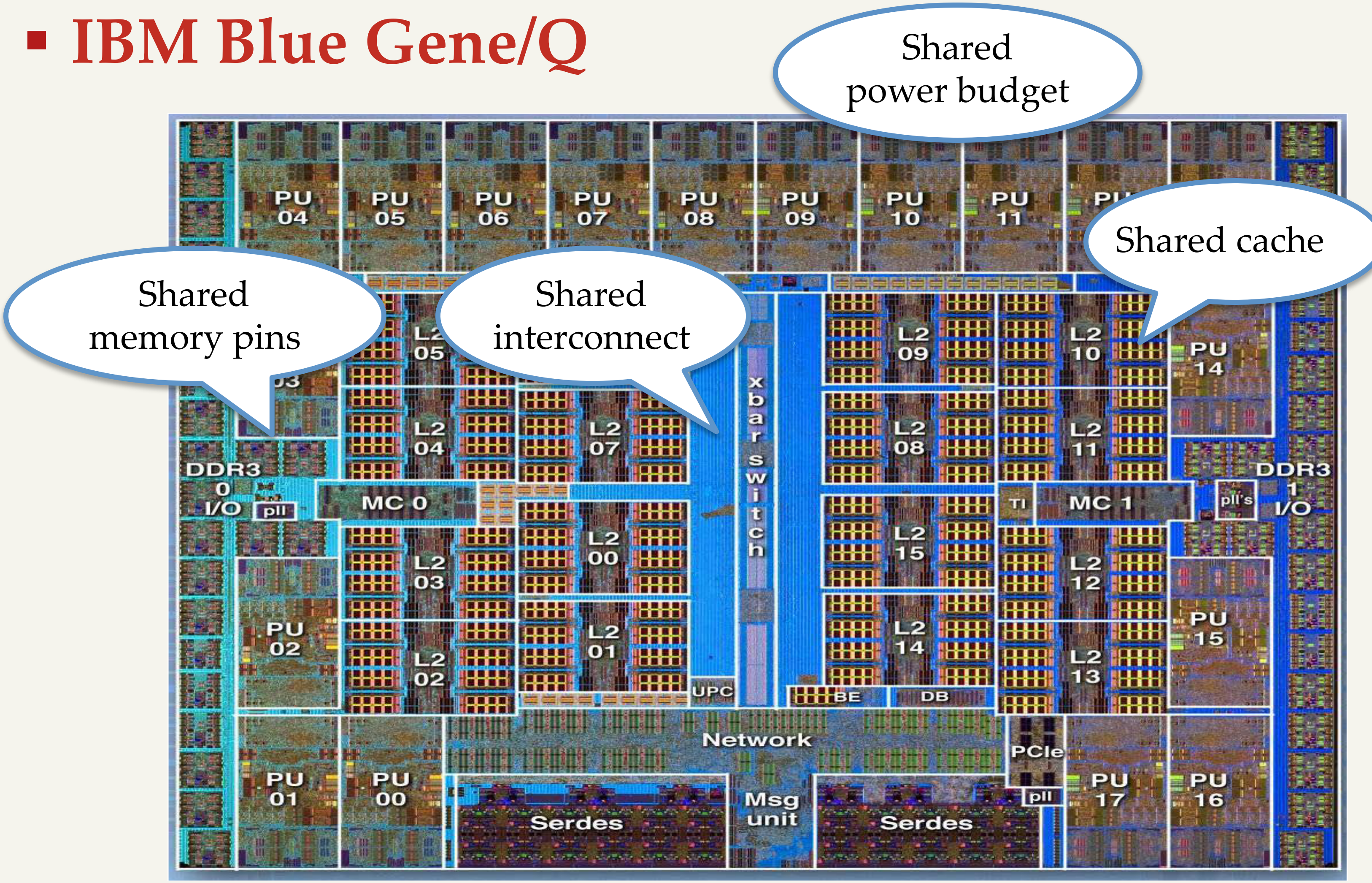
Shared On-chip resources

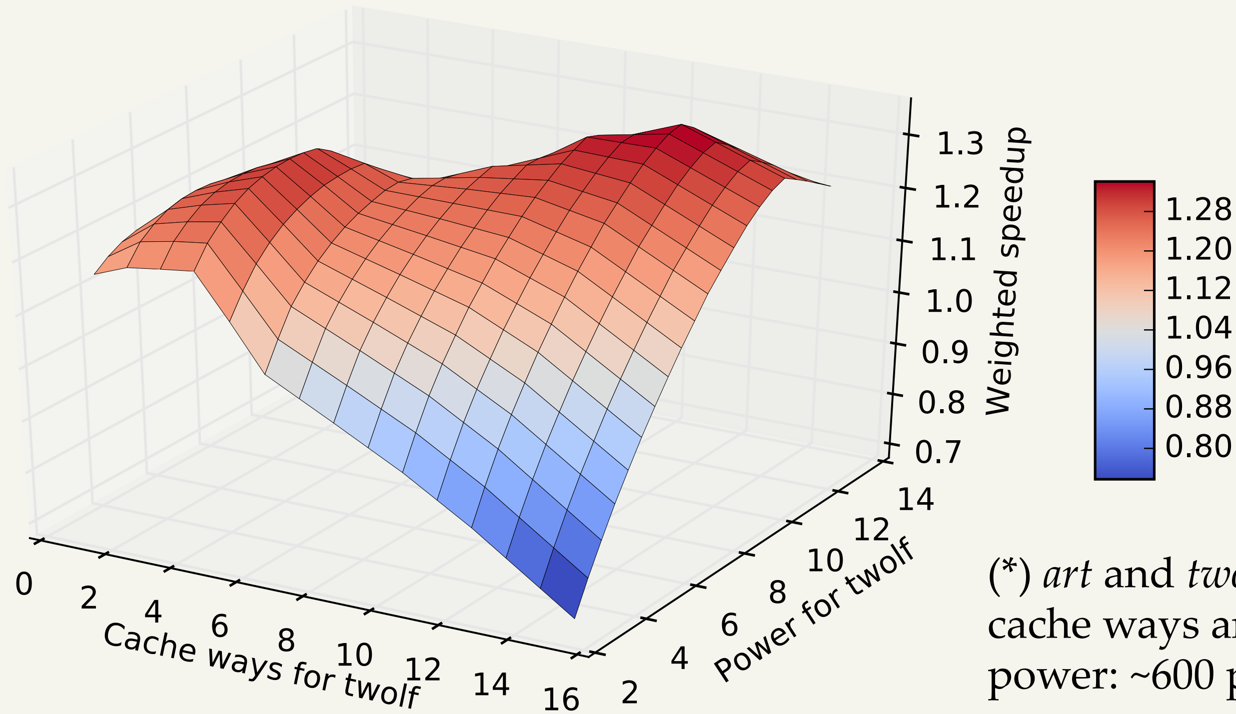
- IBM Blue Gene/Q



Source: IBM

Global allocation space very large

- Exponential increase with no. of cores, no. of resources, granularity of resources
- Global hill-climbing unlikely to scale gracefully



(*) *art* and *twolf* share 16 cache ways and 20W power: ~600 points

Cue from real-life: Markets

- **Trade off global optimality with simplicity**
  - Global optimum very expensive
  - **Market: Relatively simple, distributed mechanism**
    - » **(1) reasonably good; (2) potential for scalability**

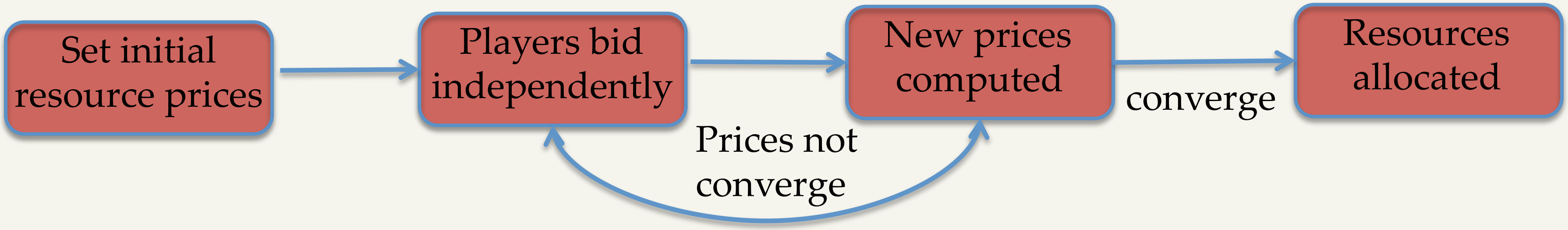
Source: Wikipedia

XChange (Wang & Martínez, HPCA 2015)

- **Effective market-based approach**
- **Market players (cores)**
  - Endowed w/ finite budgets to purchase resources
  - Goal: Maximize own utility—regardless of others
  - **Naturally concurrent**
- **Market maker**
  - Reconciles bids, posts new prices based on the supply
  - **Very fast and simple**
- **Converge to market equilibrium dynamically**
  - **High performance**: 21% average gain in throughput
  - **Scalability**: < 0.5% overhead for up to 128 cores

DYNAMIC PRICE DISCOVERY

CSL



- **Market side**
  - Prices are set in proportional to players' bids

$$price_j = \frac{\sum_i bid_{ij}}{total\_resource_j} \qquad resource_{ij} = \frac{bid_{ij}}{price_j}$$

- **Player side (cores)**
  - Model utility—resource relationship
  - Bid optimally within its budget constraint to maximize its utility

- **Budget assignment depends on the definition of optimality**
  - **Fairness-oriented**: Give same budget to everyone (XChange-EqualBudget)
  - **Performance-oriented**: Assigning budgets in proportional to the performance gap between minimum and maximum allocation (XChange-Balanced)

XChange: Open Questions

- **Optimality of market equilibrium?**
  - Pareto optimality ≠ Global optimality
  - Tragedy of commons
  - **Performance/fairness bounds?**

- **Performance vs. fairness trade-off?**
  - XChange suggests budget can function as “knob”
    - » Equal budget = “fairness-oriented”
    - » Utility-proportional budget = “performance-oriented”
  - **Control performance/fairness meaningfully?**

Proposal: ReBudget

- **Theoretic lower bounds with respect to global optimum**
  - System efficiency (performance)
  - Fairness

- **Budget assignment mechanism**
  - Builds on top of XChange
  - Control efficiency vs. fairness systematically
  - Evaluation: ReBudget >> theoretic bound often

$$Eff = \sum_i U_i(r_i)$$

- **Price of Anarchy (PoA)[2]**
  - A "superpower" determines the resource allocation that maximize overall efficiency: Eff(OPT)
  - PoA measures the cost of distributed market mechanism over global optimum:

$$PoA = \frac{Eff(Market)}{Eff(OPT)}$$

  - PoA is <u>lower</u> bound: Worst-case system efficiency
    - » Market *at least as good as* PoA (alt., can be *as bad as* PoA)

[2] Papadimitriou, *Algorithms, games, and the Internet*, STOC 2004.

PRICE OF ANARCHY

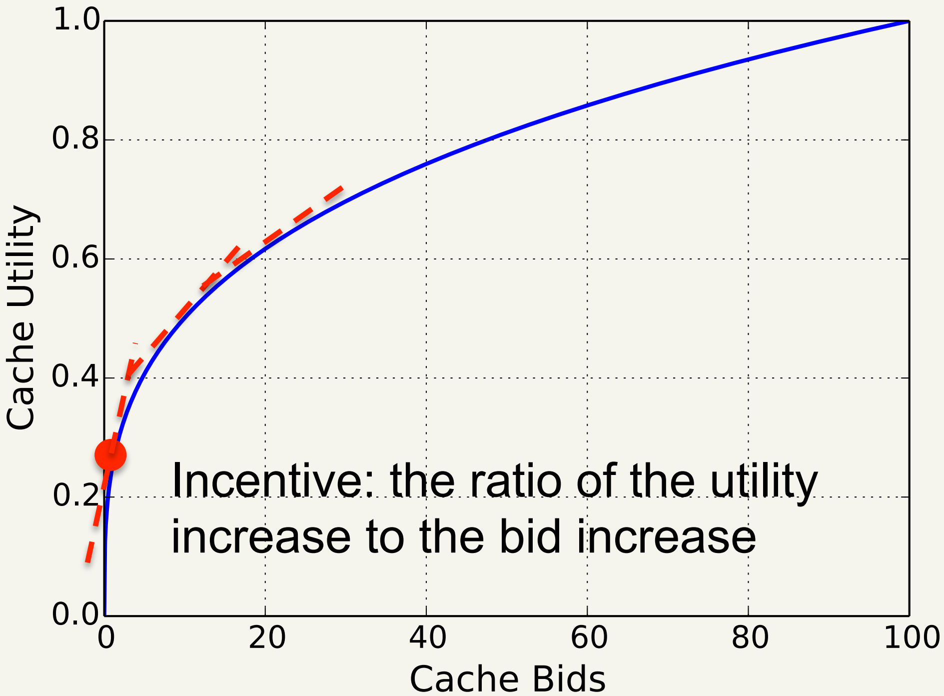
- **How "tight" can PoA bound be?**
  - The best-known theoretic bound of efficiency loss for a market under some specific conditions:[3] PoA is 75% of global optimum

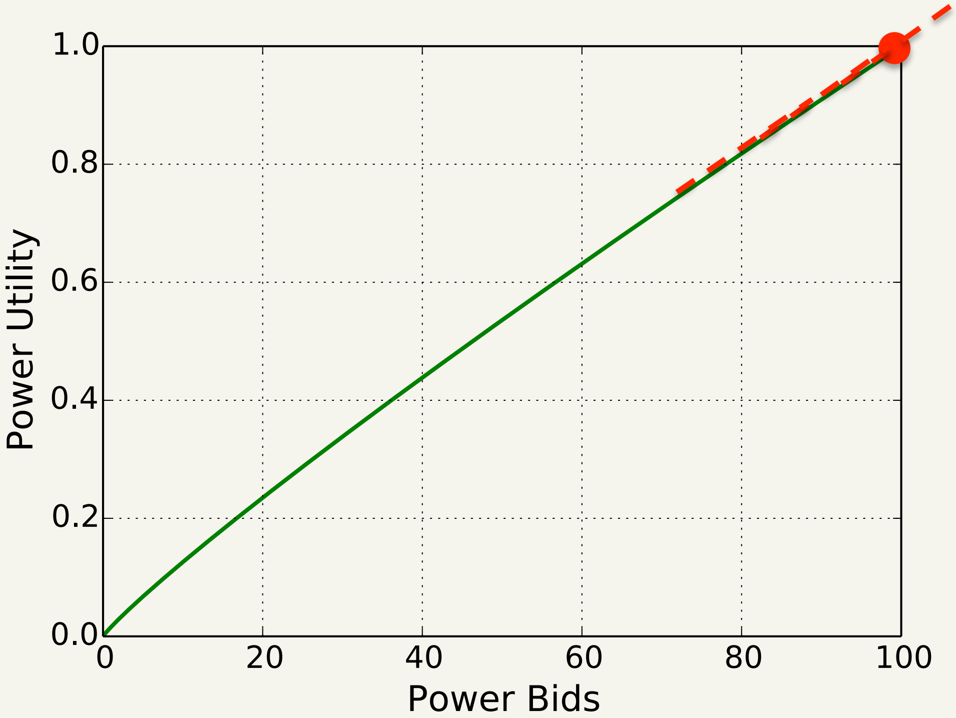
- **How close to theoretic limit can XChange be?**
  - Hypothesis: We can tighten PoA in XChange by adjusting players' budgets relative to each other

[3] Ramesh Johari and John N Tsitsiklis. Efficiency loss in a network resource allocation game, Mathematics of Operations Research, 29(3):407–435, 2004.

MARKET-BASED APPROACH

- **Market players (cores)**
  - Have finite budgets B_i to purchase resources in the system
  - Try to maximize their own utility by bidding resources





$\text{Utility} = \text{Utility}_{cache} + \text{Utility}_{power}$

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- **Player's incentive to adjust bids**
  - Intuition: if a player bids optimally, its "incentive" for different resources are the same
  - Otherwise, a revision of its bids can improve its utility
- **Mathematical representation**
  - Assume utility functions are smooth and concave*
  - Define player i's marginal utility (incentive) for resource j: $\lambda_{ij} = \frac{\partial U_i}{\partial b_{ij}}$, and incentive of player i: $\lambda_i = \max_j \lambda_{ij}$
  - Lagrange multiplier method:

$$\lambda_{ij} = \frac{\partial U_i}{\partial b_{ij}} \begin{cases} = \lambda_i & \text{if} \quad b_{ij} > 0 \\ < \lambda_i & \text{if} \quad b_{ij} = 0 \end{cases}$$

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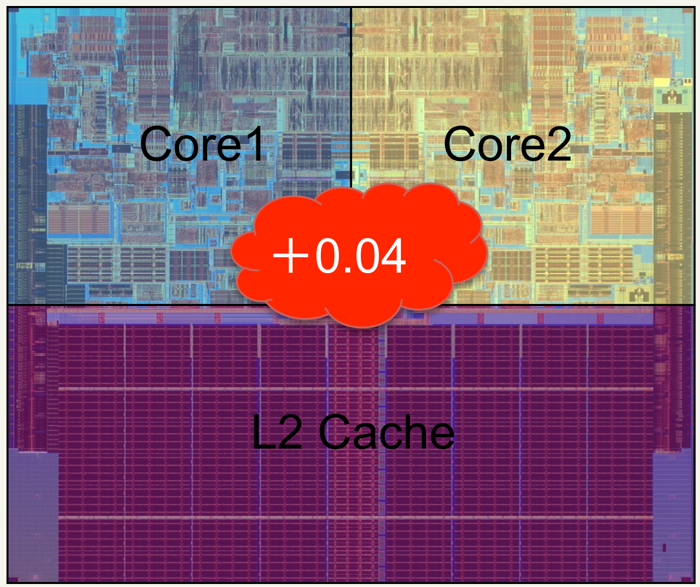
- **Intuition: at equilibrium, player i's incentive λ_i measures its utility improvement if it is given more money (i.e., assigned a larger budget)**
  - Consider 2-player market:

$99

U_1=90%

λ1=2%



$100

U_2=65%

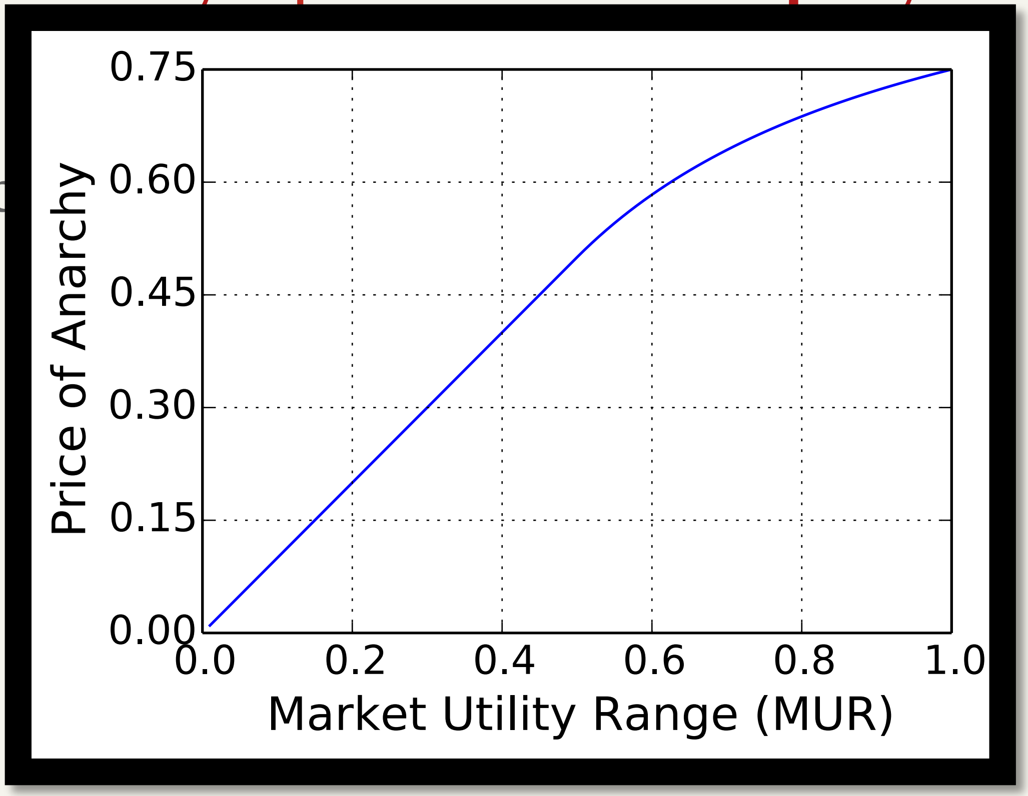
λ2=5%

- **Market utility range (MUR): maximum variation of marginal utility λ_i of all market players:**

  - Introduc

If $MUR \geq 0.5$

If $MUR < 0.5$



- **Proof**

[3] Ramesh Johari and John N Tsitsiklis. Efficiency loss in a network resource allocation game, Mathematics of Operations Research, 29(3):407–435, 2004.

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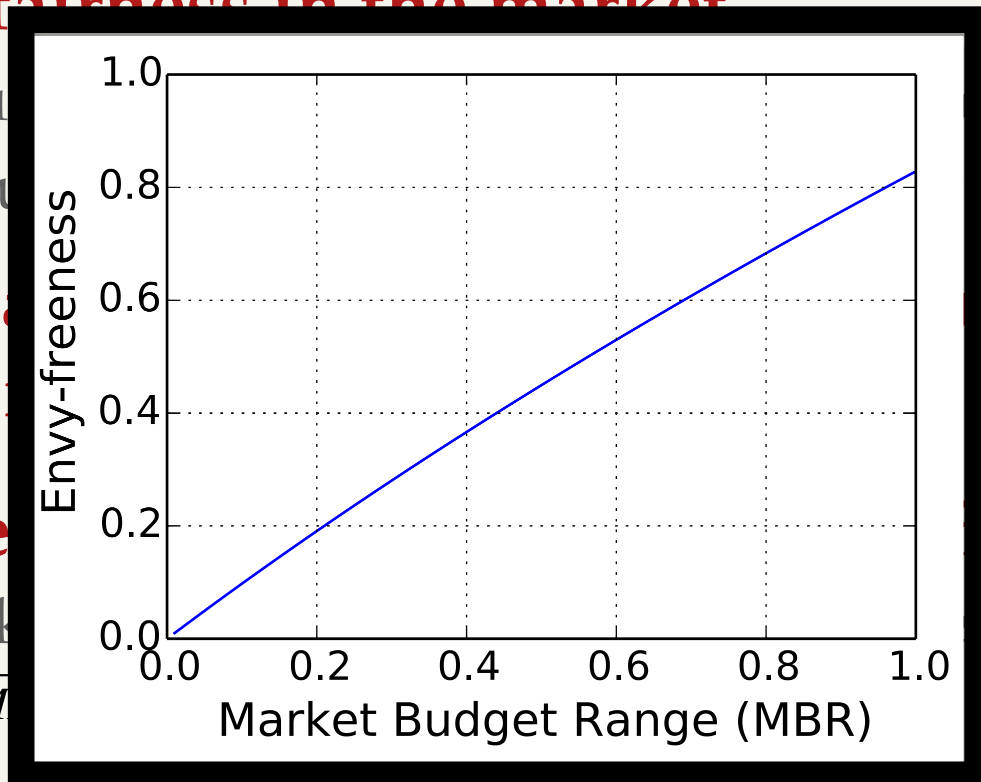
- **System fairness: Envy-freeness**
  - Envy: how a player prefer others resources: $EF_i(r) = \dfrac{U_i(r_i)}{\max_j U_i(r_j)}$
  - Given allocation $\mathbf{r} = (\mathbf{r}_1, \mathbf{r}_2, \ldots)$, envy-freeness is defined:

$$EF(r) = \min_i EF_i = \min_{i,j} \frac{U_i(r_i)}{U_i(r_j)}$$

  - C-approximate envy-free: $EF(r) \geq C$

- **Players' budget difference is a natural way to measure f**
  - Equal bu … r
  - Larger bu … ness
- **Define M … s the variation**
- **Introduce … [4]:**
  - The mark … MBR is $(2\sqrt{1+M}$



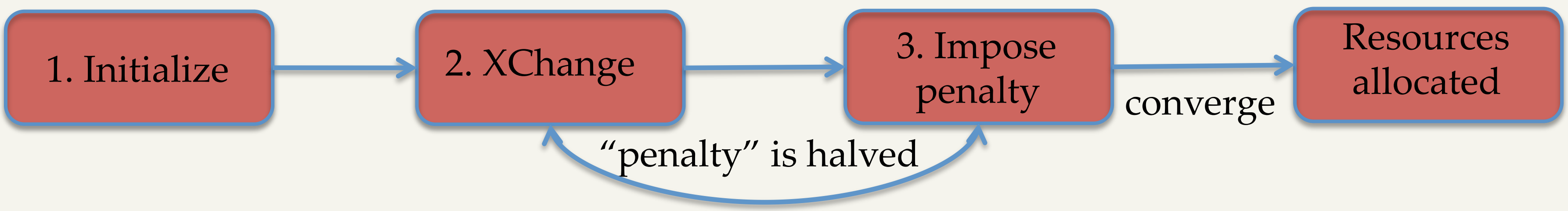
[4] Li Zhang. The efficiency and fairness of a fixed budget resource allocation game. In *Automata, Languages and Programming*, pages 485–496. Springer, 2005.

- **Measuring MUR and MBR, we can quantify the loss in efficiency and fairness of market equilibrium**

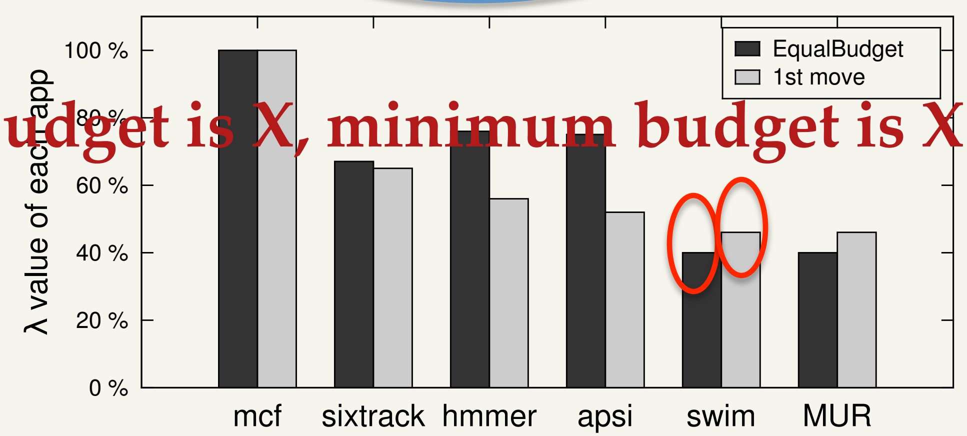
- **Use MUR and MBR to guide budget re-assignment**
  - To improve PoA: punish the players whose incentives are low
  - Guarantee minimum fairness set by system admin

REBUDGET: TRADE OFF EFFICIENCY AND FAIRNESS

- **Given a fairness target**
  1. Compute MBR: $EF = 2\sqrt{1+MBR} - 2$; initialize "penalty": $(1-MBR)\cdot\frac{X}{2}$
  2. Use XChange to reach market equilibrium
  3. Player i with incentive lower than 50% of the maximum incentive has to pay a "penalty" (equivalent to a budget cut)

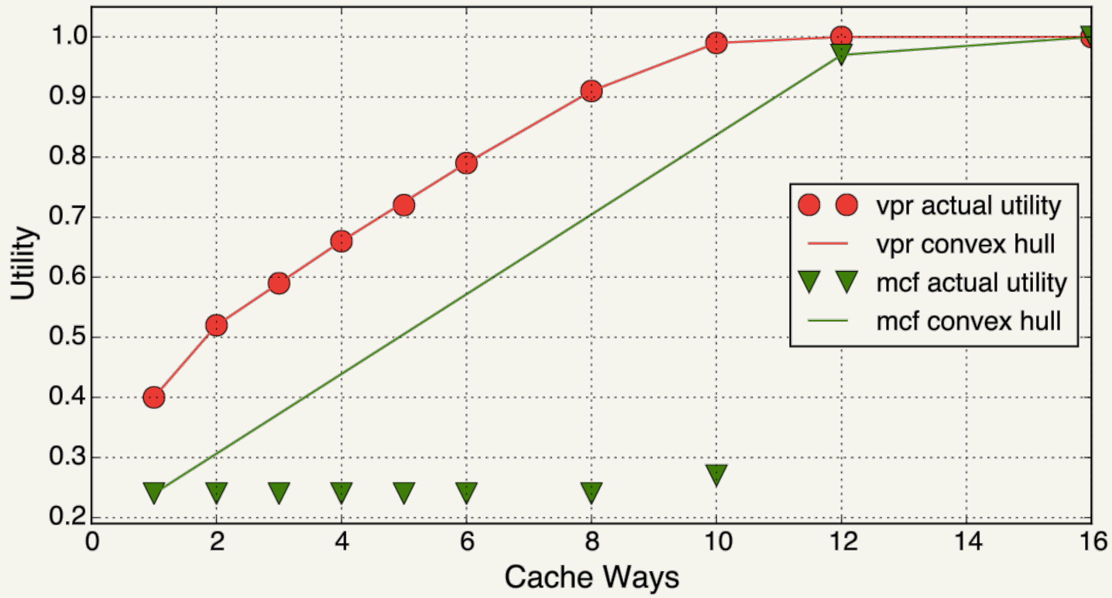


- **Maximum Budget is X, minimum budget is X•MBR**



OTHER ISSUES

- **Assumption: a player's utility function is non-decreasing, continuous, and concave**
  - Adopt Talus[4] and Futility Scaling[5], which convexifies the cache behavior



[4] Beckmann and Sanchez, HPCA, 2015.
[5] Wang and Chen, MICRO 2014

EXPERIMENTAL SETUP

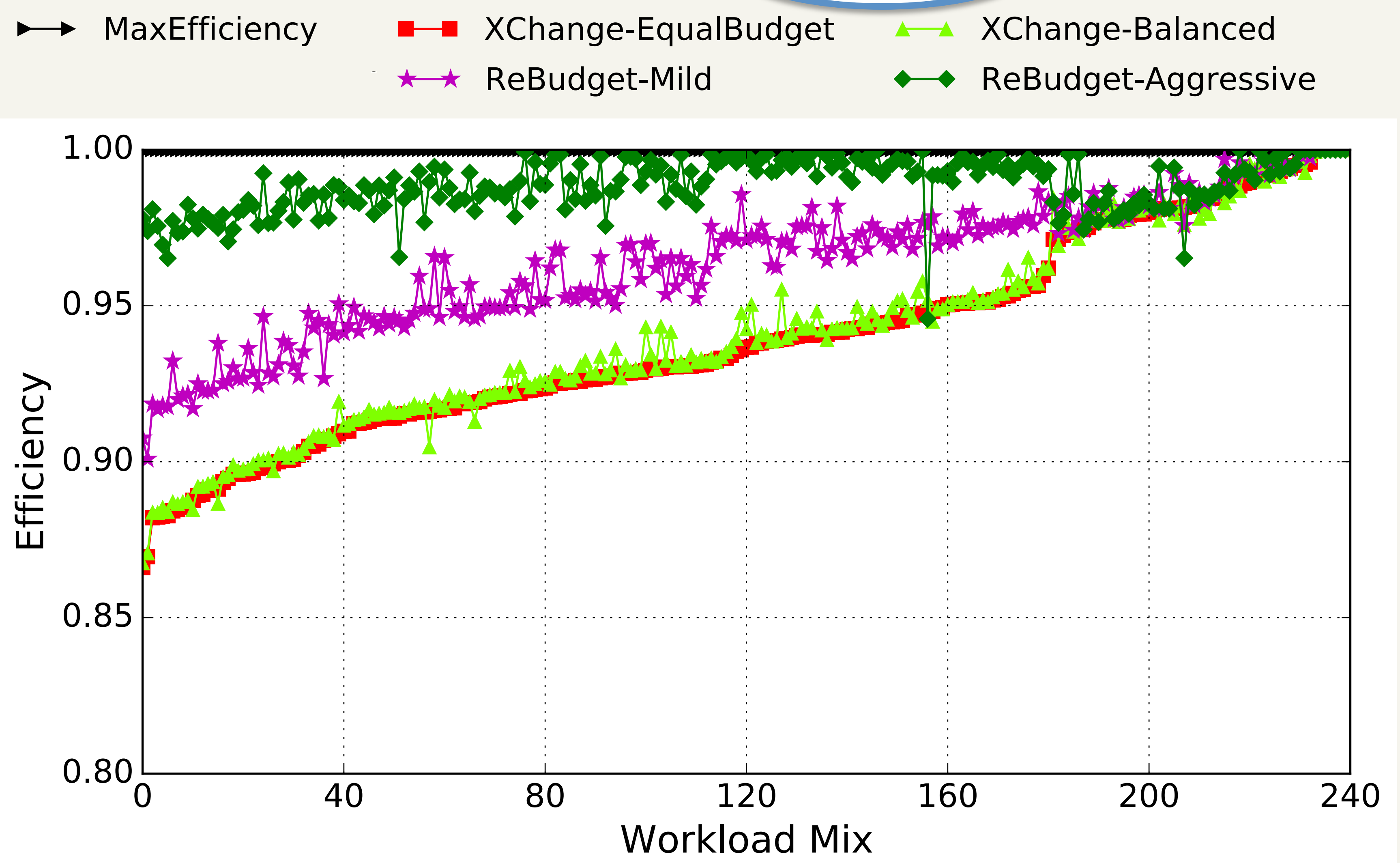
- **Simulation setup**
  - 4 GHz 4-way OoO core, 32 kB i/d L1
  - 8- and 64-core CMP
  - 4MB (16 way) and 32MB (32 way) L2 Cache
  - 10W per core as equal-share
  - DDR3-1600 channels, 4 ranks ea., 8 banks/rank

- **Performance analysis**
  - Mix of SPEC2000 and SPEC2006 multi-programed workloads

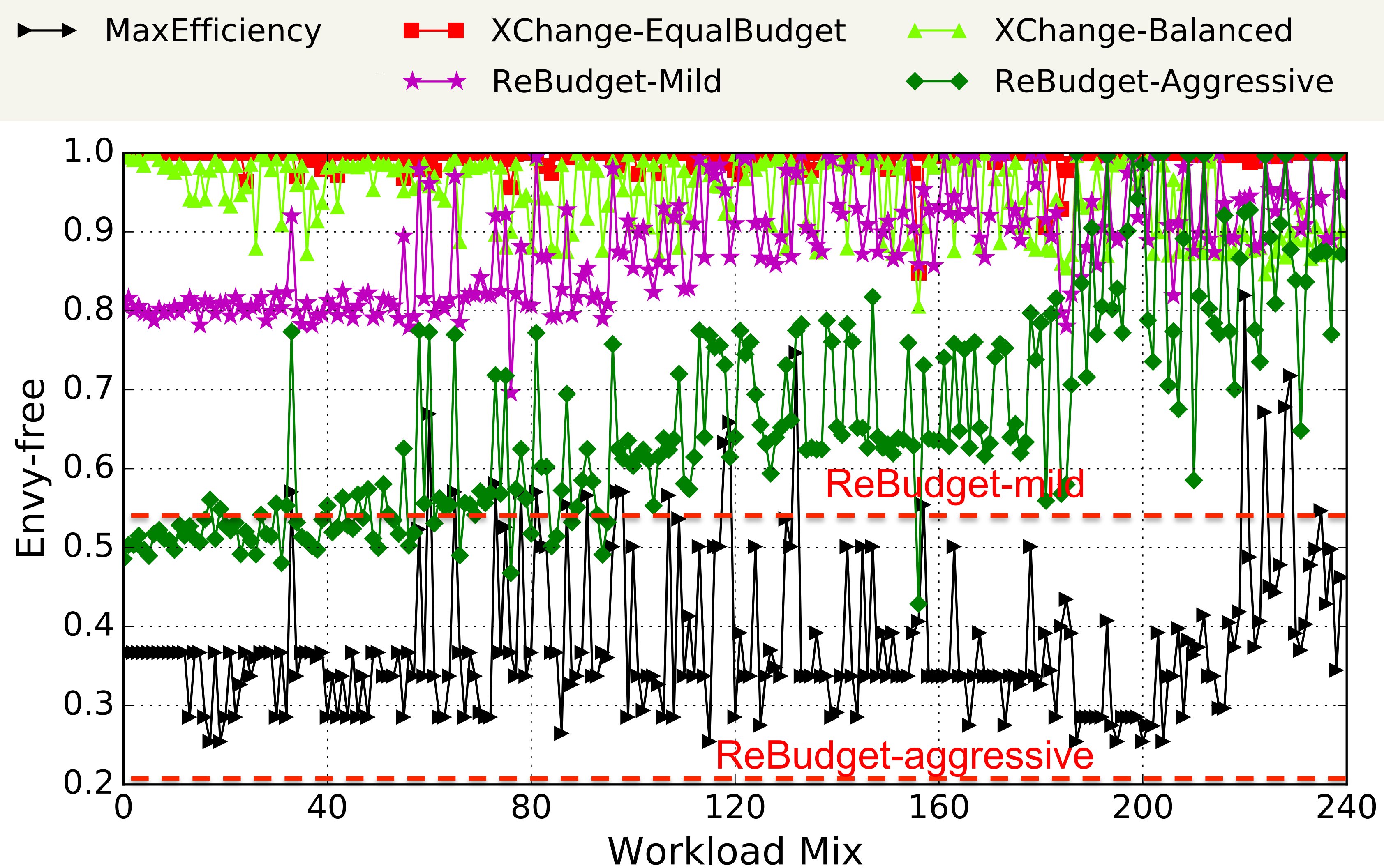
- **Analytical experiment**
  - Profile 24 SPEC 2000 and SPEC 2006 applications
  - Assume cache and power behavior are perfectly continuous and concave
  - Create 240 bundles that fall into six categories: CPBN, CCPP, CPBB, BBNN, BBPN, and BBCN
- **Implement ReBudget in simulator**
  - Utility function is modeled in the run-time
  - ReBudget is triggered every 1ms to adapt to application phase changes

EFFICIENCY

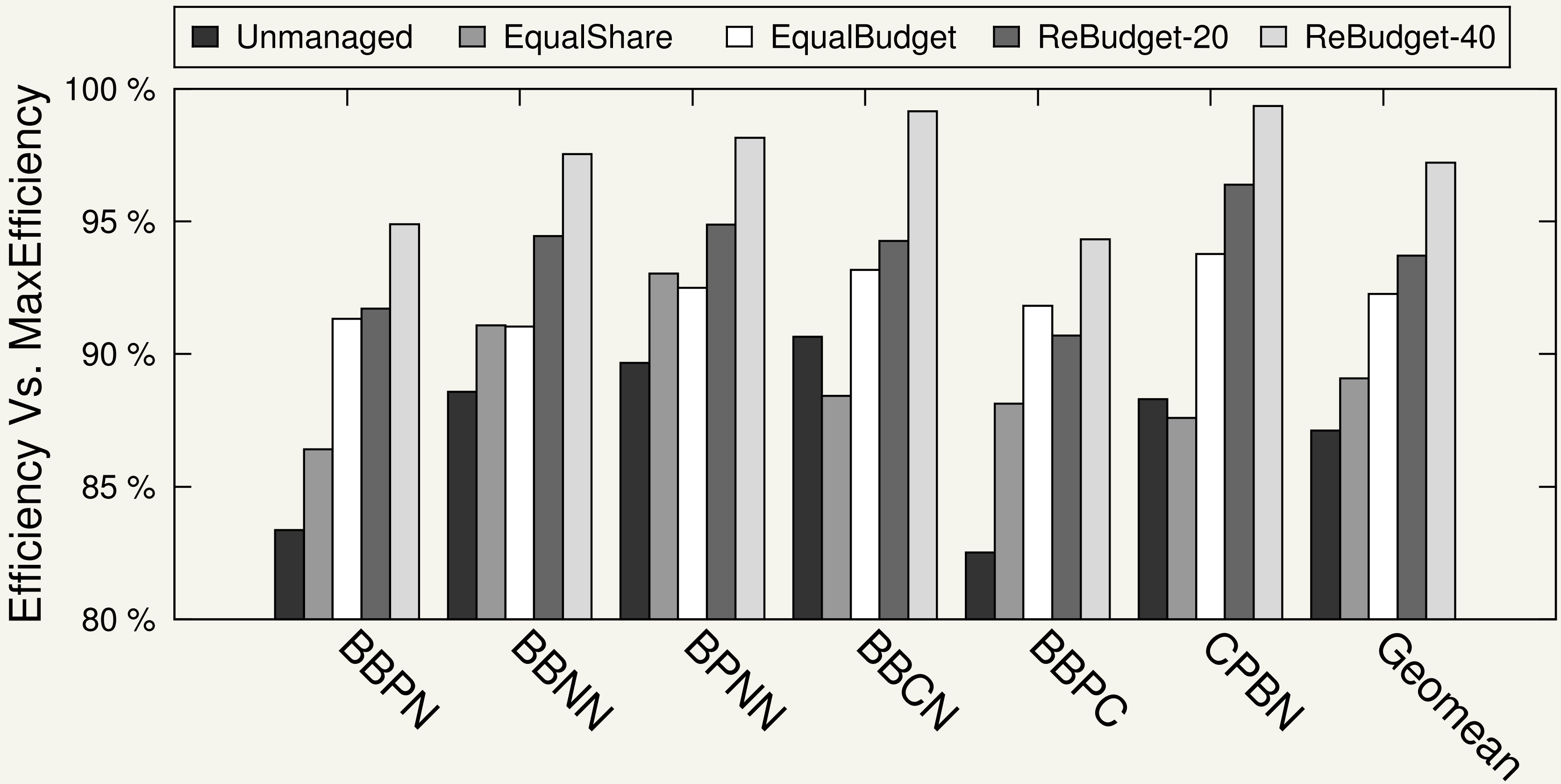
XChange, Wang and Martínez, HPCA 2015



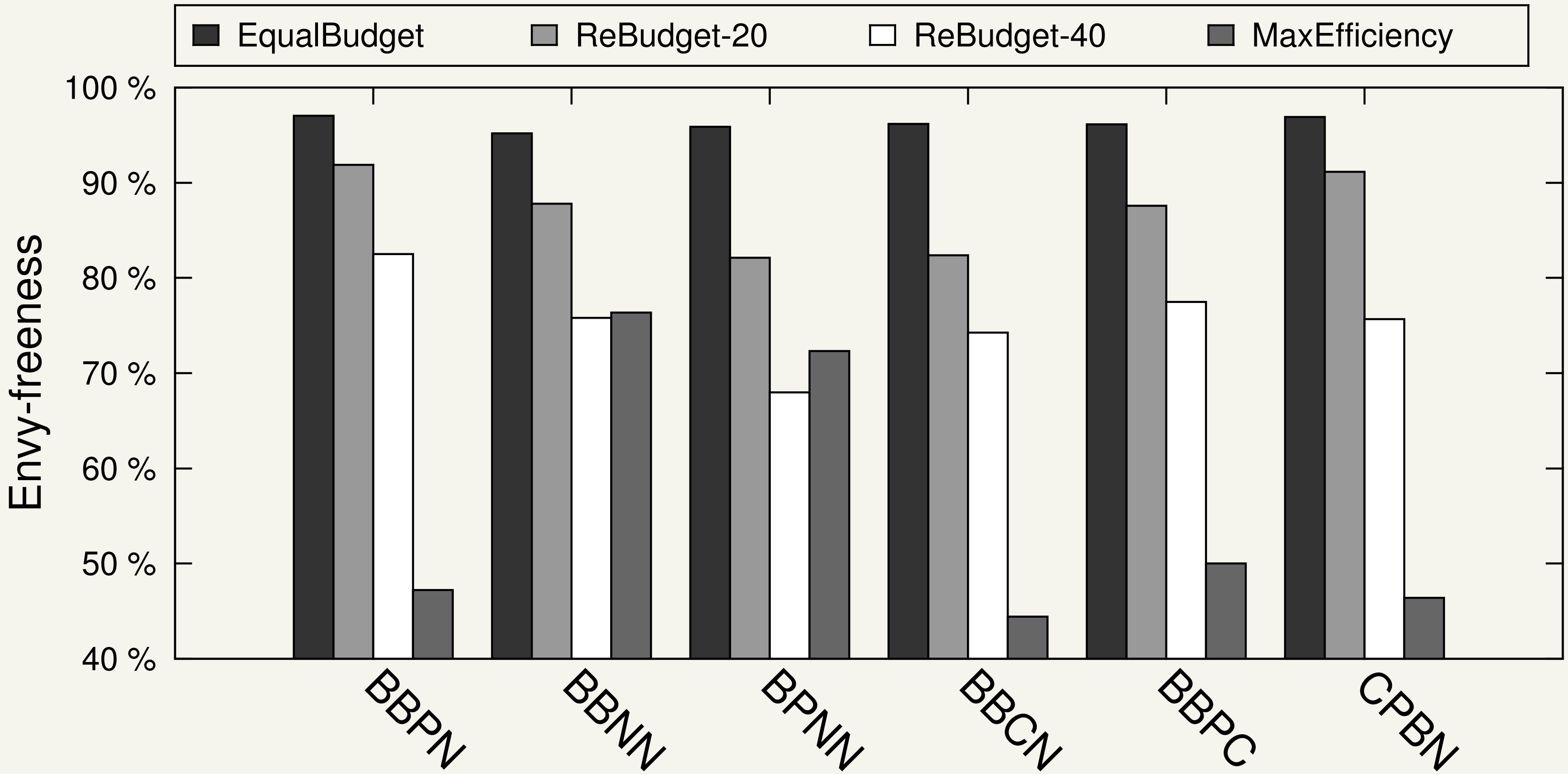
FAIRNESS


MaxEfficiency
XChange-EqualBudget
XChange-Balanced
ReBudget-Mild
ReBudget-Aggressive
Envy-free
Workload Mix
ReBudget-mild
ReBudget-aggressive
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
40
80
120
160
200
240

SIMULATION RESULTS: EFFICIENCY



SIMULATION RESULTS: FAIRNESS



SUMMARY

CSL

- **Heuristic-based market-based approach can suffer low efficiency: tragedy of commons**
- **We provide theoretic guarantees on system efficiency and fairness for arbitrary budget assignment**
  - Introduce two metrics: market utility range (MUR) and market budget range (MBR) to quantify the loss of system throughput and fairness
- **ReBudget efficiently trade off system efficiency and fairness**
  - ReBudget-aggressive achieves 95% efficiency for all application bundles

Cornell University
Computer Systems Laboratory

REBUDGET:

TRADING OFF EFFICIENCY VS. FAIRNESS IN MARKET-BASED MULTICORE RESOURCE ALLOCATION

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Backup slides