Subset-Codebook Precoding for 1-bit Massive Multiuser MIMO

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***Abstract*—We consider a massive multiuser MIMO downlink scenario with 1-bit digital-to-analog converters. QPSK information symbols are transmitted to the users by a channel-dependent mapping of the symbols to 1-bit DAC signals that are transmitted from each base station (BS) antenna. The mapping is selected to emulate the desired information bearing QPSK symbols at the users. Non-linear precoding is considered, where the mapping from the information symbols to the transmit antenna signals is selected based on the combination of information symbols. First, a linear quantized precoder is used to create tentative transmit symbols. For each combination of information symbols, a subset of antennas is selected, and an exhaustive search over a limited size codebook of precoding alternatives is performed. The resulting method significantly lowers the error floor caused by quantization, with a complexity that is linear in the number of transmit antennas.**

I. Introduction

Massive multiuser (MU) multiple-input multiple-output (MIMO) is a promising technology for fifth generation (5G) mobile communication systems [1], [2]. In extremes, massive MIMO base stations (BS) would be equipped with hundreds or thousands of antennas, and tens or hundreds of users would be served per cell, leading to potentially significant improvements in spectral and energy efficiency of wireless communication.

Increasing the number of BS antennas increases the cost of radio frequency (RF) hardware, such as power amplifiers (PAs), analog-to-digital converters (ADCs) and digital-to-analog converters (DACs), which start to dominate BS costs. Power consumption, as well as the cost of fronthaul communication between digital base band and RF units also become an issue. To be viable, massive MU-MIMO BSs have to be built with low-cost and power-efficient RF components.

Reducing the number of quantization bits in ADCs and DACs is a straightforward method to reduce hardware costs. In addition to reducing the costs of these components themselves, low-precision ADCs and DACs lower the quality requirements on all surrounding RF hardware. In particular, linearity requirements of the PAs are reduced, thus making them cheaper and more energy-efficient. Quantization also directly reduces fronthaul data rates. In the extreme, 1-bit ADCs and DACs may be considered. Such reduction in accuracy has to be carefully analyzed with respect to the resulting loss in control over the communication signals, and the corresponding loss in communication performance and reliability.

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Uplink MU-MIMO with low-precision ADCs has been thoroughly investigated in [3]–[5], and a degree of maturity has been reached. For the downlink, however, less is known about the impact of low precision DACs. The use of linear precoders followed by direct quantization [6] is shown to provide rather reliable transmission for large antenna arrays even when only 1-bit quantization is used in the DACs [7]–[9]. Such linear-quantized methods are, however, marred by error floors that are caused by the quantization operation, which produce heavy distortions on the received signal constellations. These distortions increase with the number of users, and their effect becomes significant when the signal-to-noise ratio (SNR) increases.

Recently, non-linear precoding methods have been proposed to mitigate these distortions. In [10], [11], non-linear precoding was used when QPSK constellations are used to communicate with the users, and in [12] when higher order modulation constellations are transmitted to the receivers. These non-linear methods significantly outperform 1-bit linear-quantization methods when the ratio of users to antennas is high. In [11], [12], the inherent integer programming problem is relaxed to convex problems. Convex solvers with polynomial complexity in the number B of Tx antennas and the number U of users can be applied. The resulting complexity is still high, e.g., in the semi-definite relaxation approach of [11], the worst case complexity scales *cubically* in $2B$.

In this paper, we consider codebook-based methods to reduce the complexity of non-linear precoders in 1-bit massive MU-MIMO systems. We find approximate solutions to the integer program directly by searching over a limited set of precoding alternatives. The search set reduction is based on first carrying out a linear-quantized precoder, and selecting a subset of antennas that the search concentrates on. Complexity is further reduced by searching only over a limited size codebook in the selected subset, not over all possibilities. Outside the selected set, a linear quantized precoder is used. The codebook is selected so that the search can be performed with extremely low complexity. As a result, the complexity of non-linear precoding only grows *linearly* in B.

II. System Model

We consider a downlink multiuser MIMO system, where there are U single-antenna users that are served by a base station with B transmitter antennas. The channels between the base station transmitter (Tx) and the users (Rx) are described by a single-tap channel matrix $\mathbf{H}$ of dimensions $U \times B$.

The RF chains in the BS have limited complexity, so that there is a 1-bit DAC for the in phase and quadrature component at each antenna. Without loss of generality, this is interpreted so that the output of each Tx antenna is a QPSK-symbol, i.e., taking values in the set $\mathcal{X}_\mathrm{Q} = \{\pm 1 \pm \mathrm{i}\}/\sqrt{2}$. The transmitted signal is thus a $B \times 1$ vector $\mathbf{x} \in \mathcal{X}_\mathrm{Q}^B$, and the received (Rx) signals at the users are

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}\,, \tag{1}$$

where $\mathbf{n}$ is a $U \times 1$ vector of complex-valued additive noise, assumed Gaussian. We shall model the entries in $\mathbf{H}$ as i.i.d. circularly symmetric complex Gaussians with unit variance. The channel gains and transmit powers are absorbed to the variance of the entries of $\mathbf{n}$. Accordingly, the variance of n_u is $\frac{B}{\gamma_u U}$, where the signal-to-noise ratio (SNR) γ_u is the total received power at user u, divided with the number of users, and the noise power spectral density at u.

Nonlinear precoding is applied at the Tx. Information transmitted to the users is mapped to the $U \times 1$ vector $\mathbf{s} \in \mathcal{X}_\mathrm{Q}^U$ of QPSK-symbols, one for each user. It is assumed that $\mathbf{H}$ is perfectly known at the Tx. Protocols for approximately achieving this from uplink transmissions can be designed even if the uplink base station receiver has limited accuracy ADCs.

The task of the nonlinear precoder is to find a vector quantization function $\mathbf{q_H}$ yielding the transmitted signal $\mathbf{x} = \mathbf{q_H}(\mathbf{s})$ such that the information can be reliably decoded at the individual users. The quantizer depends on the channel $\mathbf{H}$. The Rx signals can be modeled as

$$\mathbf{y} = \mathrm{diag}(\mathbf{v})\,\mathbf{s} + \tilde{\mathbf{n}}\,, \tag{2}$$

where the vector $\mathbf{v}$ consists of an effective complex-valued channel to each user, and the interference and distortion caused by the multiuser transmission and the quantization at Tx is added to the noise in $\tilde{\mathbf{n}}$. Note that both $\mathbf{v}$ and the statistics of $\tilde{\mathbf{n}}$ depend on the quantization map $\mathbf{q_H}$. The contribution of the quantizer to v_n constitutes a user-specific complex-valued generalization of the scaling factor β in [11].

At the user u, the transmitted information symbol x_u is estimated from y_u. We assume that the receiver knows the effective channel gain v_u from estimation. Using this, the users may first find

$$z_u = \frac{v_u^* y_u}{|v_u|^2}\,, \tag{3}$$

from which hard decision estimates of the transmitted symbols can be found as

$$\hat{s}_u = \mathrm{sign}_\mathbb{C}\left(z_u\right). \tag{4}$$

Here $\mathrm{sign}_\mathbb{C}()$ takes the sign of both the real and the imaginary parts of its argument and normalizes with $\sqrt{2}$, and thus returns a QPSK symbol. A diagram of the considered 1-bit massive MU-MIMO system is shown in Fig. 1 .

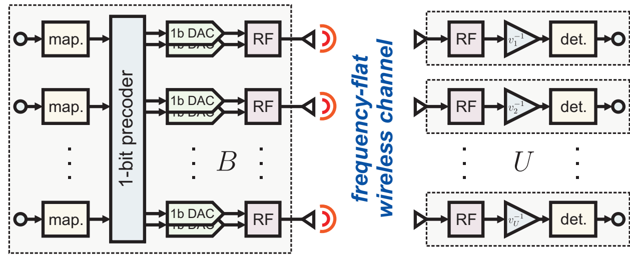

Fig. 1. Tx- and Rx-chains in the considered 1-bit precoded system.

We decouple the quantizer optimizations for different signal constellations $\mathbf{s}$ by assuming that $\mathbf{v}$ is known when selecting the quantizer $\mathbf{q_H}(\mathbf{s})$.

To proceed, we need a metric for non-linear precoding. In [7], [11], [12], the mean square error (MSE) is used as a quality metric. As argued in [10], for a low-cardinality constellation such as QPSK, a metric taking the structure of the constellation into account may be more accurate, as certain large distortions do not cause errors. Accordingly, we take the quality metric to be average multiuser QPSK bit-error-rate (BER), which has a well-known expression in terms of distances to decision surfaces.

In contrast to [10], where the distances of the multiple users were combined in a product metric, we consider the smallest distance as a multiuser metric. This is optimal at high SNR, where the minimum distance dominates performance. With a given quantizer $\mathbf{q_H}$ and signal $\mathbf{s}$, the scaled smallest distance of the received signal of user u from its decision surfaces is

$$d(z_u, s_u) = \min{}_\mathbb{C}\{s_u^* z_u\}\,, \tag{5}$$

where $\min{}_\mathbb{C}(w) \equiv \min\{\mathrm{Re}\, w, \mathrm{Im}\, w\}$. The received signal has been rotated according to the phase of the received mean constellation v_u, so that Rx u assumes z_u to be a conventional QPSK symbol. In (5), this symbol is rotated to the first quadrant, and the distances to the decision surfaces are given by the real and imaginary parts. If z_u is on the wrong side of the decision surface, the distance becomes negative. For a given transmitted signal $\mathbf{s}$ we thus consider the multiuser distance

$$d_m(\mathbf{s}|\mathbf{q}) = \min_u d(z_u, s_u)\,, \tag{6}$$

which is the minimum distance to the decision surface over the set of users. The quantizer for the signal $\mathbf{s}$ is then

$$\mathbf{q}_B(\mathbf{s}|\mathbf{v}) = \arg\max_{\mathbf{q}\in\tilde{\mathcal{Q}}} d_m(\mathbf{s}|\mathbf{q})\,. \tag{7}$$

optimized over the set $\tilde{\mathcal{Q}}$ of single signal precoders with cardinality 4^B.

In principle, for each combination $\mathbf{s}$ of transmitted symbols, a different mapping $\mathbf{q}$ is thus selected. Each mapping selects one out of 4^B alternate output signals. From (1) it is however clear that there is a $\mathbb{Z}_4$ symmetry; the same mapping is optimal for any of the four symbol vectors $\mathrm{i}^m \mathbf{s}$, for integers $m = 0, \ldots, 3$. Without loss of generality, it is thus sufficient to consider mappings for signals $\mathbf{s} \in \mathcal{S}$, where e.g., s_1 is be fixed. Altogether $|\mathcal{S}| = 4^{U-1}$ precoder mappings are thus needed, and the cardinality of the precoder set is

$$|\mathcal{Q}| = 4^{B+U-1} . \tag{8}$$

The optimization space is excessively large even for moderate values of B. For each candidate precoder, received signals have to be estimated according to (1), and metrics have to be calculated in (6). All of this should happen within channel coherence time, for a transient user population.

Accordingly, heuristic methods to reduce the set $\mathcal{Q}$ have been considered in the literature. Direct quantization of zero forcing (ZF) precoders [8], [9] produces a $\mathbf{q}$ without any search. In [7], improved MMSE precoders are considered, where quantization noise is taken into account when finding the linear precoder.

Here, we shall consider codebook-based methods to reduce $\mathcal{Q}$. For this, we consider codebooks with codeword length D and cardinality N,

$$\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_N\} , \tag{9}$$

where the codewords have fourth-root-of-unity entries, $\mathbf{c}_n \in \mathcal{R}_4^D$ with $\mathcal{R}_4 = \{1, \mathrm{i}, -1, -\mathrm{i}\}$.

III. Subset Codeword Selection Precoding

In *subset codeword selection* (SCS) precoding, we have a fixed codebook $\mathcal{C}$ and a fixed subset selection principle, which do not depend on the channel realization $\mathbf{H}$. Precoding happens as follows. First, for each signal constellation $\mathbf{s} \in \mathcal{S}$, we take a baseline precoder $\tilde{\mathbf{q}}(\mathbf{s}) \in \mathcal{X}_{\mathrm{Q}}^B$. Using these baseline precoders, we get baseline Rx signal constellations, and take the means of those constellations, rotated to the appropriate quadrant, as the effective channels $\mathbf{v}$. This way we have decoupled the optimization problems for different $\mathbf{s}$ from each other, and can use (7) online for each $\mathbf{s}$ that is needed.

When optimizing the quantizer for a given $\mathbf{s}$, a subset $\mathcal{D}$ of Tx antennas with cardinality D is selected. The restriction of the baseline precoder to this set is $\tilde{\mathbf{q}}_{\mathcal{D}}$. We restrict the search for quantization maps for this $\mathbf{s}$ such that the precoder is fixed to $\tilde{\mathbf{q}}(\mathbf{s})$ outside $\mathcal{D}$, and in $\mathcal{D}$, the candidates in the set

$$\mathcal{Q}_{\mathbf{s}} = \{\mathbf{q}_{\mathcal{D}} \mid \mathbf{q}_{\mathcal{D}} = \tilde{\mathbf{q}}_{\mathcal{D}} \circ \mathbf{c};\ \mathbf{c} \in \mathcal{C}\} \tag{10}$$

are considered. Here $\circ$ is the element-wise (Hadamard) product of two vectors. Note that the Hadamard-rotation of any $\mathbf{s} \in \mathcal{X}_{\mathrm{Q}}^D$ with a $c_n \in \mathcal{R}_4^D$ remains in $\mathcal{X}_{\mathrm{Q}}^D$.

We shall use two kinds of baseline precoders. The 1-bit quantized matched filter (MF) precoder is

$$\tilde{\mathbf{q}}_{\mathrm{mf}} = \mathrm{sign}_{\mathbb{C}} \left(\mathbf{H}^{\mathrm{H}} \mathbf{s}\right) , \tag{11}$$

whereas the 1-bit quantized ZF precoder is

$$\tilde{\mathbf{q}}_{\mathrm{zf}} = \mathrm{sign}_{\mathbb{C}} \left(\mathbf{H}^{\mathrm{H}} \left(\mathbf{H}\mathbf{H}^{\mathrm{H}}\right)^{-1} \mathbf{s}\right) . \tag{12}$$

For ZF precoding, the $U \times U$ covariance matrix $\mathbf{H}\mathbf{H}^{\mathrm{H}}$ has to be inverted. We denote the antenna-specific outputs by the baseline precoders as $\tilde{\mathbf{x}} = \tilde{\mathbf{q}}(\mathbf{s})$

The underlying assumption is that the baseline precoders are largely good, and that critical mis-quantizations of the baseline precoders can be corrected with a limited (and hence low complexity) search. To apply SDS on top of these baseline precoders, one thus has to develop a method for selecting the subset $\mathcal{D}$, and one has to design a codebook $\mathbf{C}$.

We use the following heuristic for selection of subsets. The processed signals z_u of (3) at the users have a contribution from each transmit antenna. Denoting the channel between the BS and u by $\mathbf{h}_u^{\mathrm{H}}$, and omitting the noise contribution, we have

$$z_u \approx \mathbf{h}_u^{\mathrm{H}} \mathbf{x} / v_u = \sum_{n=1}^{N} h_{un} x_n / v_u \equiv \sum_{n=1}^{N} z_{un}(x_n) , \tag{13}$$

where we stress the (linear) dependence of z_{un} on x_n. Now, according to (5), each antenna contributes towards the decision distance of user with a factor $\min_{\mathbb{C}}\{s_u^* z_{un}(x_n)\}$. For antenna n, quantizers differ by outputting different symbols x_n. We define the improvement metric for user u on antenna n as

$$a_{un} = \max_{x \in \mathcal{X}_{\mathrm{Q}}} \left[\min{}_{\mathbb{C}}\ s_u^* z_{un}(x) - \min{}_{\mathbb{C}}\ s_u^* z_{un}(\tilde{x}_n)\right]_+ , \tag{14}$$

that is, the maximum improvement in minimum distance for this user, if the signal on antenna x_n were changed. If the baseline $\tilde{x}_n$ is the best signal for this user on this antenna, $a_{un} = 0$. Summing these, we get an improvement metric for antenna n:

$$A_n = \sum_u a_{un} . \tag{15}$$

The D antennas with largest A_n are selected as the subset $\mathcal{D}$.

IV. Codebooks

A priori it is not clear what characterizes a good codebook for the problem. First, the assumption that the baseline precoders are largely good indicates that $\mathcal{C}$ has to include the all ones codeword $\mathbf{c}_0$. Furthermore, from the structure of the baseline precoders (11,12), and (1), it is clear that no codeword in the discrete D-dimensional torus $\mathcal{T}_D = \mathcal{R}_4^D$ can de directly excluded. The cardinality of $\mathcal{T}_D$ may, however, be too large for practical use. For example, if a $D = 8$ dimensional subset of Tx antennas would be selected, $|\mathcal{T}_D| = 65536$, which may lead to prohibitive complexity.

Codebooks that are proper subsets of $\mathcal{T}_D$ thus have to be designed. The codebook design problem becomes a source coding problem on $\mathcal{T}_D$. The source is distributed as the optimum subset quantizer for a given statistics $f(\mathbf{H})$ of the channel. The distribution of this source over $\mathcal{T}_D$ is not known. Constructing a proper distortion metric would involve integration of the distortion over $f(\mathbf{H})$. From (1) it follows that the set of precoders live in a linear space. A natural prior distance metric between codewords is the Euclidean distance

$$d(\mathbf{c}_n, \mathbf{c}_m) = \|\mathbf{c}_n - \mathbf{c}_m\|_2 , \tag{16}$$

which is the square root of the Hamming distance of a binary representation of the codebook, so that the Hamming distance can be directly used as well. Here, we use this metric for the design of codebooks. Also, from (1) it follows that there is continuity in $\mathcal{T}_D$, so that the distortion measure at neighboring points are similar.

For one user, 1-bit quantized MF and ZF precoders are equivalent, and they provide optimal quantizations. Also, as can be seen in [11], at low SNR, linear-quantized MF and ZF perform well, and cannot be improved. For a low number of users, ZF performs reasonably well up to moderate SNRs. This indicates that for low SNR, and a low number of users, codewords in $\mathcal{T}_D$ close to $\mathbf{c}_0$ are more likely to be optimal, whereas for high number of users and/or high SNR, the optimal codeword may be more evenly distributed over $\mathcal{T}_D$.

If it is enough to search over small changes to the baseline precoder in the D-dimensional subset $\mathcal{D}$, corresponding codebooks would be *minimum-maximum distance* (Min-Max-Dist) ones. One would select N distinct codewords at minimum distance from $\mathbf{c}_0$, corresponding to rotating one or a few antennas with $\pm\mathrm{i}$, or with -1. It is a straight forward combinatorical exercise to construct Min-Max-Dist codebooks with low cardinality.

A more conventional *maximum-minimum distance* (Max-Min-Dist) codebook would have an even spread of codewords over $\mathcal{T}_D$. Such codebooks are based on an assumption that the optimum codeword is i.i.d. in $\mathcal{T}_D$. Constructing Max-Min-Dist codebooks is more challenging than constructing Min-Max ones. We use results from the literature on codebooks for Grassmannian subspace packings with finite alphabets [13], [14]. First, we observe that $\mathcal{T}_D$ can be decomposed as the direct product $\mathcal{T}_D = \mathcal{R}_4 \times \widetilde{\mathcal{T}}_D$, where the $\mathcal{R}_4$ indicates the overall phase of a vector, and the Grassmannian lines $\widetilde{\mathcal{T}}_D$ are cosets w.r.t. $\mathcal{R}_4$, i.e., vectors in $\mathcal{T}_D$ that are equivalent up to overall rotations. The rotations in $\mathcal{R}_4$ acting on any $c \in \widetilde{\mathcal{T}}_D$ generate antipodal points, i.e., points at maximum distance in $\mathcal{T}_D$. Thus structured codebooks with large distances can be constructed from direct products of $\mathcal{R}_4$ and Grassmannian codebooks, which are subsets of $\mathcal{T}_D$. For the latter, we use codebooks based on Mutually Unbiased Bases (MUBs). In dimensions that are powers of 2, such codebooks can be generated with entries in $\mathcal{R}_4$ [13], [14], some of which are optimal packings [13]. It should be noted that here we generate codes for the linear space $\mathcal{T}_4$ based on Grassmannian codes. In projective geometry, selection of coset representative for codebooks in $\widetilde{\mathcal{T}}_D$ (i.e., which of the vectors $\mathrm{i}^m c$ is used as a codeword) is irrelevant. For use in linear space, the problem of selecting a representative is not benign, however [15].

Using structured codebooks based on MUBs has the benefit that MUBs in dimensions of 2 can be constructed from generalized Hadamard matrices [14], and thus come with an inherent possibility of using Hadamard transforms to simplify codebook processing. The dominant complexity in SCS will thus be the subset selection, which again is dominated by the multiplicative complexity of (13), *linear* in B. We leave more detailed discussions of codebooks for future work.

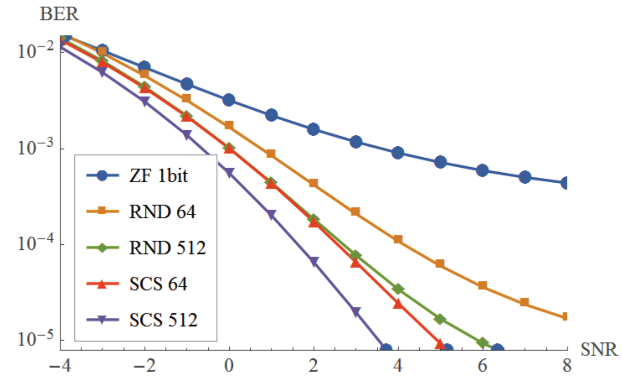

Fig. 2. Comparison of SCS and random subset selection. Max-Min-Dist codebooks of different size. $B = 32$; $U = 4$.

V. Simulation Results

We now show simulation results to demonstrate the efficacy of SCS precoding. We consider a scenario with $B = 32$ Tx antennas and SCS in subspaces of dimension $D = 8$. We assume that all users have the same SNR. All channels are i.i.d. Rayleigh fading. The baseline precoder is ZF, if not otherwise indicated. In all simulations, the channels are Monte Carlo sampled, and for each sample, the 4^{U-1} different transmit vector cosets in $\mathcal{S}$ are systematically considered, by evaluating the QPSK bit-error probability for transmissions in a given channel sample.

First, we investigate the subset selection criterion. We take $U = 4$ users and $N_s = 10^4$ channel samples. In Fig. 2, codebooks with different size, $N \in \{64, 512\}$ are used in two scenarios. Precoding in subsets selected with criterion (15) (legend "SCS N") is compared to precoding in randomly selected subsets (legend "RND N"). SCS targets the use of limited precoding to a subset where it has more impact than in a randomly selected one, and provides overall better performance both at low and high SNR than RND.

Next, in Fig. 3 Min-Max-Dist codebooks (legend "NS") are contrasted to Max-Min-Dist codebooks (legend "N") for codebook sizes $N \in \{8, 16, 32, 64\}$ and $U = 4$. There is an interesting cross-over behavior. The small-distance codebooks result in better performance at low SNR, whereas the large-distance ones are systematically better at high SNR. This reveals interesting (and unclear) characteristics of the distribution of the source signal over $\mathcal{T}_D$ as a function of SNR.

High-SNR performance of Max-Min-Dist codebooks with different N is shown in Fig. 4, for $U = 4$. The quantized Zero Forcer (legend "ZF 1bit") shows an error floor. This floor is reduced by applying SCS, and becomes lower with increasing N. Note that the error flor does not vanish with SCS, at least for small codebooks. Here $N_s = 10000$ channel samples are used, leading to approximately $2 \cdot 10^7$ different configurations of bits and channels considered in the simulation. Accordingly, the estimated BERs are rather reliable for BER $> 10^{-5}$, and ultimately unreliable at BER 10^{-7} and below. For the combination of B and U considered for this figure, the

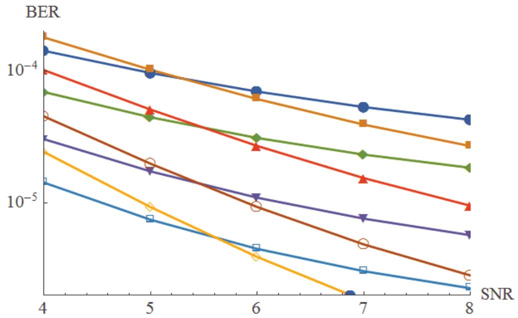

Fig. 3. Crossover behavior for SCS with Max-Min-Dist and Min-Max-Dist codebooks of different size. $B = 32$; $U = 4$.

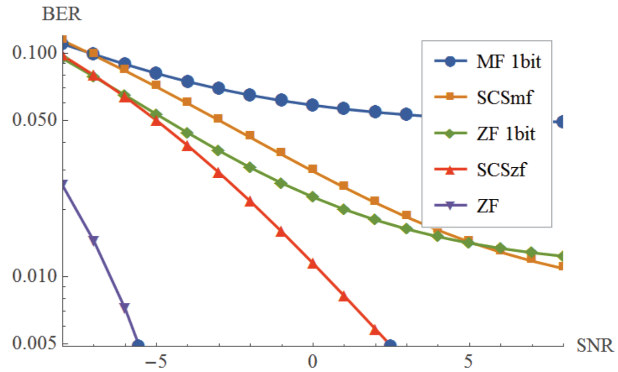

Fig. 5. BER vs. SNR for 8-user SCS. $B = 32$; $N = 64$.

lowering of error floor is visible at impractically low levels of BER.

In Fig. 5, a massive MIMO scenario with higher load is considered, with transmissions to $U = 8$ users from $B = 32$ antennas. There, SCS shows clear performance improvement already at low SNR, at BER 0.01. SCS both with MF and ZF baseline are considered, the latter being considerably better. Infinite precision ZF (legend "ZF") is also shown for comparison. The loss of SCS as compared to infinite precision ZF is 7 dB at BER 0.01.

VI. Conclusion

We have proposed a novel, nonlinear precoding method for 1-bit massive multiuser MIMO systems. Our algorithm consists of a linear-quantized precoder followed by a refinement stage where we first select a subset of the antennas with a quality metric, and then optimize the antenna symbols in the subset with a limited codebook search to reduce the error rate at the user side. The method, referred to as subset codeword selection, is computationally efficient and significantly outperforms conventional 1-bit precoders by substantially reducing their error floor at high SNR, especially in an overloaded massive MIMO system, where the number of users is relatively large. The complexity of the search over precoding can be significantly reduced by using an appropriate codebook. We find that the principle for designing the best codebooks depends on the target SNR operation point.

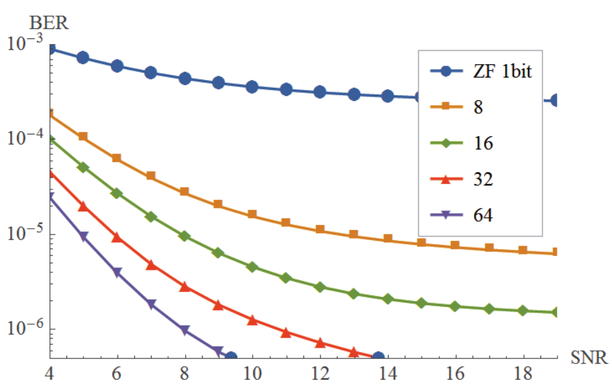

Fig. 4. Perfromance of SCS with Max-Min-Dist codebooks of different size. $B = 32$; $U = 4$.

References

[1] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, and F. Tufvesson, "Scaling up MIMO: Opportunities and challenges with very large arrays," *IEEE Sign. Proc. Mag.*, vol. 30, no. 1, pp. 40–60, Jan. 2013.

[2] E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Comm. Mag.*, vol. 52, no. 2, pp. 186–195, Feb. 2014.

[3] C. Risi, D. Persson, and E. G. Larsson, "Massive MIMO with 1-bit ADC," arxiv:1404.7736, Apr. 2014.

[4] S. Jacobsson, G. Durisi, M. Coldrey, U. Gustavsson, and C. Studer, "One-bit massive MIMO: Channel estimation and high-order modulations," in *Proc. IEEE ICC Workshops*, Jun. 2015, pp. 1304–1309.

[5] C. Studer and G. Durisi, "Quantized massive MU-MIMO-OFDM uplink," *IEEE Transactions on Communications*, vol. 64, no. 6, pp. 2387–2399, Jun. 2016.

[6] A. Mezghani, R. Ghiat, and J. A. Nossek, "Transmit processing with low resolution D/A-converters," in *Proc. IEEE Int. Conf. Electron., Circuits, Syst. (ICECS)*, Dec. 2009, pp. 683–686.

[7] O. Usman, H. Jedda, A. Mezghani, and J. Nossek, "MMSE precoder for massive MIMO using 1-bit quantization," in *Proc. IEEE ICASSP*, Mar. 2016, pp. 3381–3385.

[8] A. Saxena, I. Fijalkow, and A. Swindlehurst, "On one-bit quantized zf precoding for the multiuser massive MIMO downlink," in *Proc. IEEE Sensor Array and Multich. Sign. Proc Worksh. (SAM)*, Jul. 2016, pp. 1–5.

[9] ——, "Analysis of one-bit quantized precoding for the multiuser massive MIMO downlink," preprint arXiv:1610.06659v1, pp. 1–10, Oct. 2016.

[10] H. Jedda, J. Nossek, and A. Mezghani, "Minimum BER precoding in 1-bit massive MIMO systems," in *Proc. IEEE Sensor Array and Multich. Sign. Proc Worksh. (SAM)*, Jul. 2016, pp. 3381–3385.

[11] S. Jacobsson, G. Durisi, M. Coldrey, T. Goldstein, and C. Studer, "Quantized precoding for massive MU-MIMO," preprint arXiv:1610.07564v1, pp. 1–31, Oct. 2016.

[12] ——, "Nonlinear 1-bit precoding for massive MU-MIMO with higher-order modulation," in *Proc. Asilomar Conf.*, Oct. 2016, pp. 1–5.

[13] A. R. Calderbank, P. J. Cameron, W. M. Kantor, and J. J. Seidel, "Z_4-Kerdock codes, orthogonal spreads, and extremal Euclidean line-sets," *Proc. London Math. Soc.*, vol. 3, pp. 436–480, 1997.

[14] T. Inoue and R. W. Heath, "Kerdock codes for limited feedback MIMO systems," in *Proc. IEEE ICASSP*, Mar. 2008, pp. 3113–3116.

[15] R.-A. Pitaval and O. Tirkkonen, "Joint Grassmann-Stiefel quantization for MIMO product codebooks," *IEEE Trans. Wireless Comm.*, vol. 13, no. 1, pp. 210–222, Jan. 2014.