ECE 6775
High-Level Digital Design Automation
Fall 2024

Analysis of Algorithms

Cornell University

CSL

Announcements

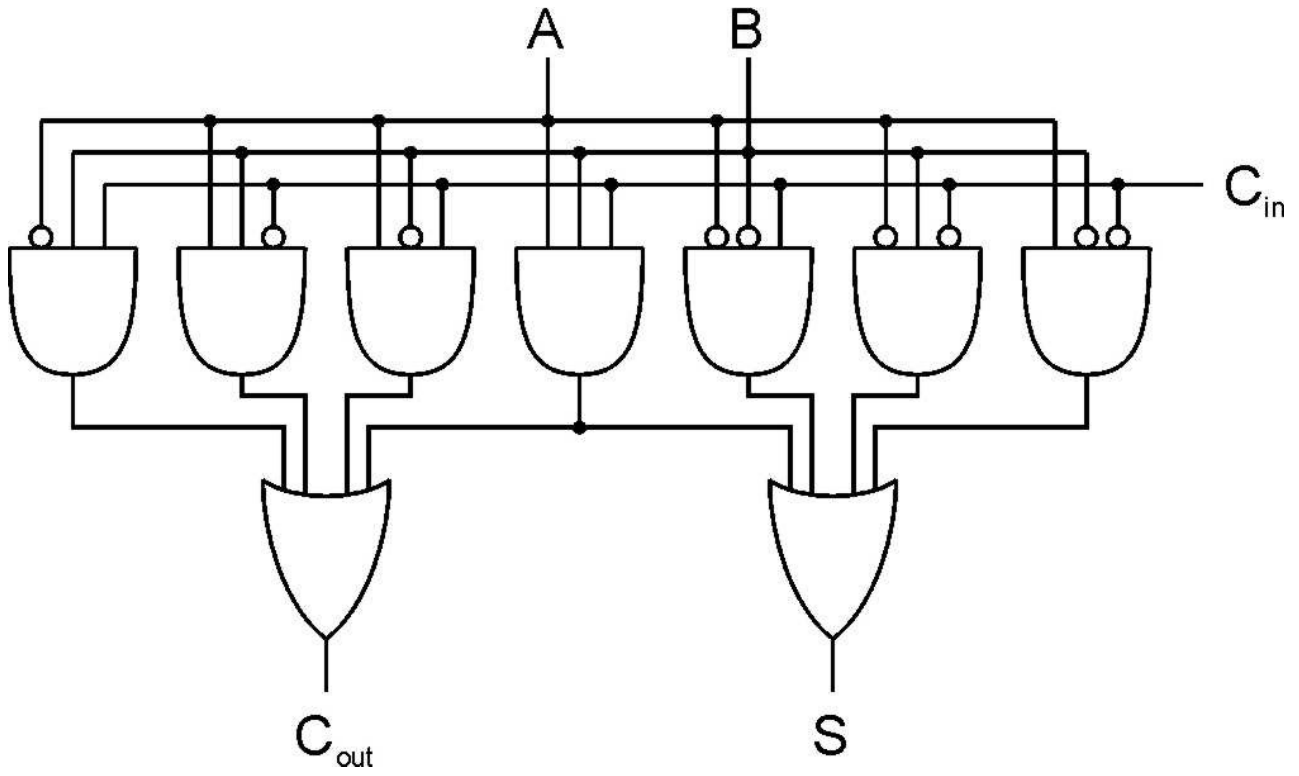
- Lab 1 due tomorrow
- HW 1 will be released today
- Instructor office hours rescheduled to **Thursdays 5-6pm**, starting today

Review: LUT Mapping

(1) How many 3-input LUTs are needed to implement the following full adder?

(2) How about using 4-input LUTs?

A	B	C_{in}	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

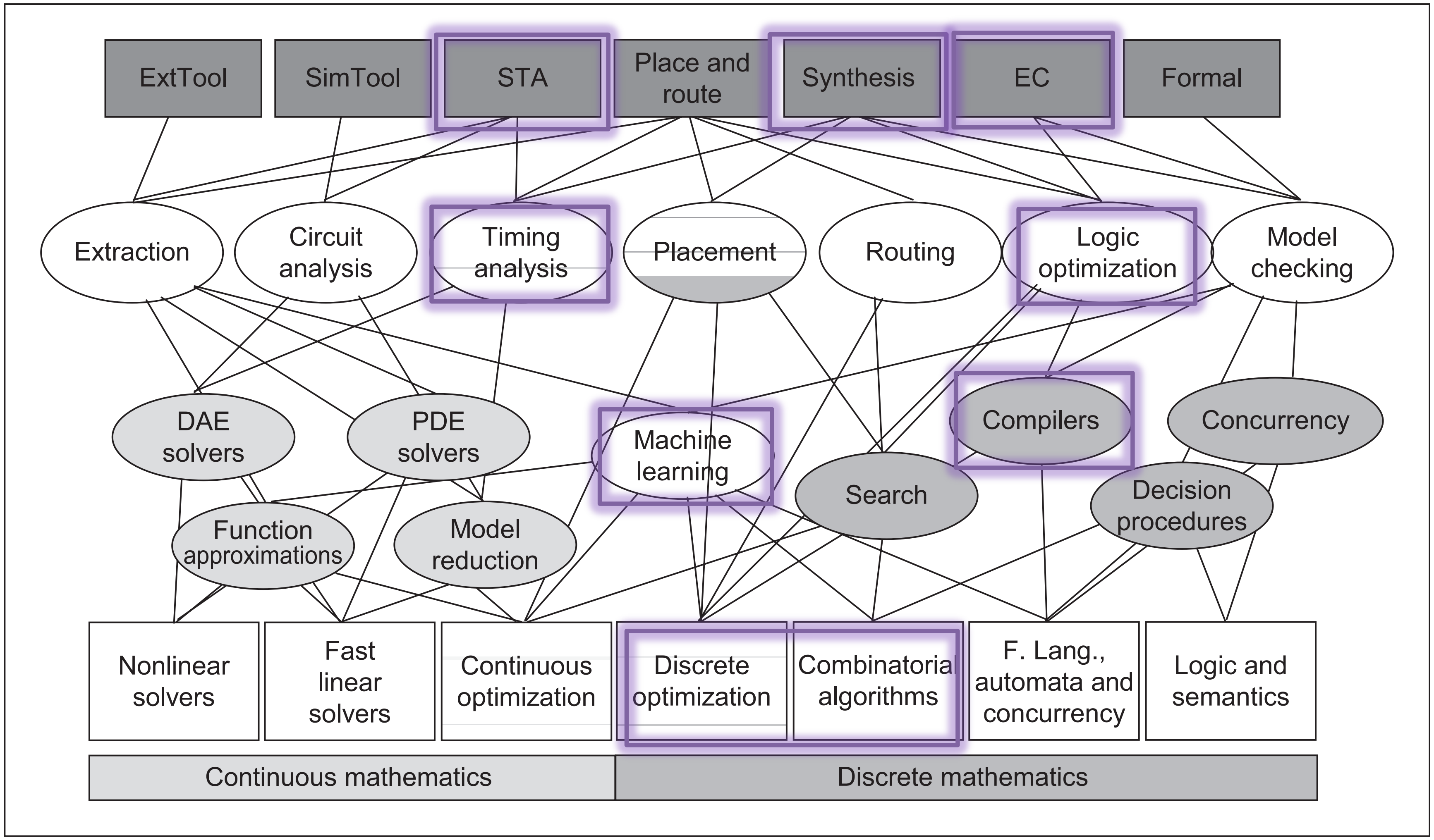


Agenda

- Basics of algorithm analysis
  - Complexity analysis and asymptotic notations
  - Taxonomy of algorithms
- Basics of graph algorithms
  - An EDA application: static timing analysis

Recap: Algorithms Drive Automation

Topics touched on in 6775



Key Algorithms in EDA

[source: Andreas Kuehlmann, Synopsys Inc.]

Analysis of Algorithms

- Need a systematic way to compare two algorithms
  - Execution time is typically the most common criterion used
  - Space (memory) usage is also important in most cases
  - But difficult to compare in practice since these algorithms may be implemented on different machines, use different languages, etc.
  - Plus, execution time is usually input-dependent

- **big-O** notation is widely used for asymptotic analysis
  - Complexity is represented with respect to some natural & abstract measure of the problem size *N*

Big-O Notation

- Express execution time as a function of input size *n*
  - Running time F(n) is of order G(n), written as F(n) is **O**(G(n)) when $\exists n_0, \forall n \geq n_0, F(n) \leq K \cdot G(n)$ for some constant *K*
  - F will not grow larger than G by more than a constant factor
  - G is often called an "**upper bound**" for F
- Interested in the worst-case input & the growth rate for large input size

Big-O Notation (cont.)

- How to determine the order of a function?
  - Ignore lower order terms
  - Ignore multiplicative constants
  - Examples:

    $3n^2 + 6n + 2$ is $O(n^2)$

    $n^{1.1} + 1000n$ is $O(n^{1.1})$, $n^{1.1}$ is also $O(n^2)$

    $n! > C^n > n^C > \log n > \log \log n > C$

    $\Rightarrow n! > n^{10} > n^2 > n \log n > n > \log n$

- What do asymptotic notations mean in practice?
  - If algorithm A is $O(n^2)$ and algorithm B is $O(n \log n)$,

    <u>we usually say algorithm B is **more scalable**.</u>

More Asymptotic Notions

- **big-Omega** notation: $F(n)$ is $\Omega(G(n))$
  - $\exists n_0, \forall n \geq n_0, F(n) \geq K \cdot g(n)$ for some constant K
    G is called a "**lower bound**" for F
- **big-Theta** notation: $F(n)$ is $\Theta(G(n))$
  - If G is both an upper and lower bound for F, it describes the growth of a function more accurately than big-O or big-Omega
  - Examples:
    $4n^2 + 1024 = \Theta(n^2)$
    $n^3 + 4n \neq \Theta(n^2)$

Exponential Growth

- Consider a 1 GHz processor (1 ns per clock cycle) running 2^N operations (assuming each op requires one cycle)

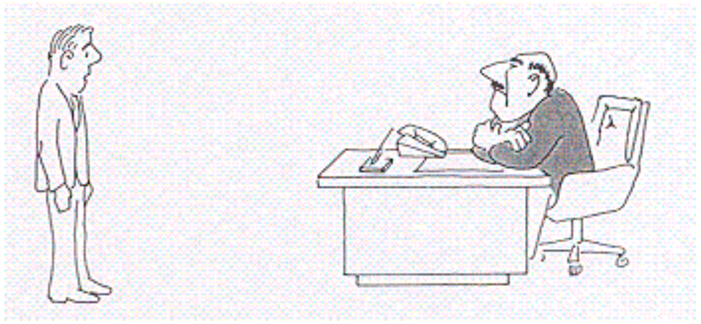
N	**2^N**	**1ns x 2^N**
10	10^3	1 us
20	10^6	1 ms
30	10^9	1 s
40	10^{12}	16.7 mins
50	10^{15}	11.6 years
60	10^{18}	31.7 years
70	10^{21}	31710 years

NP-Complete

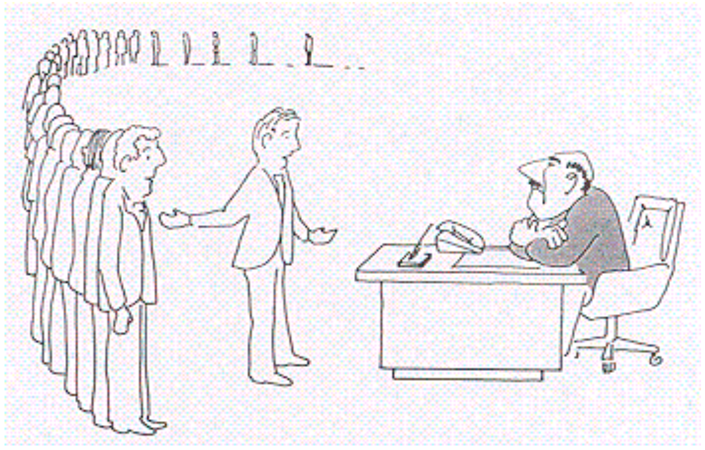
- The class **NP-complete** (NPC) is the set of decision problems which we “believe” there is no polynomial time algorithms (hardest problem in NP)
- **NP-hard** is another class of problems, which are at least as hard as the problems in NPC (also containing NPC)
- If we know a problem is in NPC or NP-hard, there is (very) little hope to solve it exactly in an efficient way

How to Identify an NP-Complete Problem

- I can't find an efficient algorithm, I guess I'm just too dumb.



- I can't find an efficient algorithm, but neither can all these famous people.



- I can't find an efficient algorithm, because no such algorithm is possible.

[source: "Computers and Intractibility" by Garey and Johnson]

More formally – In NP-completeness proofs, a **reduction** is the process of transforming one problem (which is known to be NPC) into another in polynomial time to show that solving the second problem would also solve the first, proving the second problem is at least as hard.

Problem Intractability

- Most of the nontrivial EDA problems are intractable (NP-complete or NP-hard)
  - Best-known algorithm complexities that grow exponentially with n, e.g., $O(n!)$, $O(n^n)$, and $O(2^n)$.
  - No known algorithms can ensure, in a time-efficient manner, globally optimal solution
- **Heuristic** algorithms are used to find near-optimal solutions
  - Be content with a “reasonably good” solution

Types of Algorithms

- There are many ways to categorize different types of algorithms
  - Polynomial vs. Exponential, in terms of computational effort
  - Optimal (or Exact) vs. Heuristic, in solution quality
  - Deterministic vs. Stochastic, in decision making
  - Constructive vs. Iterative, in structure

  …

Various Algorithm Design Techniques

- There can be many different algorithms for solving the same problem
  - Exhaustive search
  - Divide and conquer
  - Dynamic programming
  - Greedy
  - Linear programming (LP)
  - Integer linear programing (ILP)
  - Network flow
  - Evolutionary algorithms
  - Simulated annealing

  …

Topics touched on in 6775

Broader Classification of Algorithms

- Combinatorial algorithms
  - **Graph algorithms**

  …
- Computational mathematics
  - **Optimization algorithms**
  - Numerical algorithms

  …
- Computational science
  - Bioinformatics
  - Linguistics
  - Statistics

  …
- Digital logic
  - Boolean minimization

  …
- Information theory & signal processing

…

- Machine learning and statistical classification

Topics touched on in 6775

Many more

[source: en.wikipedia.org/wiki/List_of_algorithms]

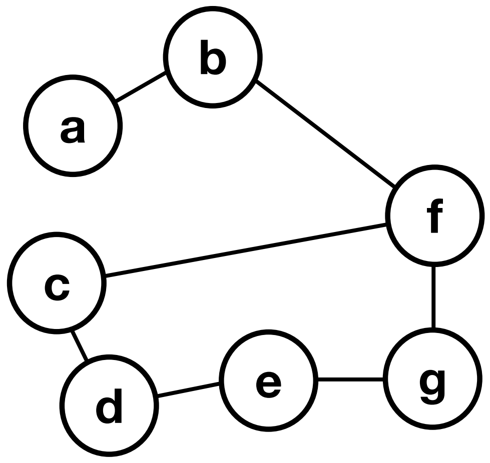
Graph Definition

- Graph: a set of objects and their connections
  - Ubiquitous: any binary relation can be represented as a graph
- Formal definition:
  - G = (V, E), V = {v_1, v_2, ..., v_n}, E = {e_1, e_2, ..., e_m}
    - V : set of **vertices** (nodes), E : set of **edges** (arcs)
  - **Undirected graph**: an edge {u, v} also implies {v, u}
  - **Directed graph**: each edge (u, v) has a direction

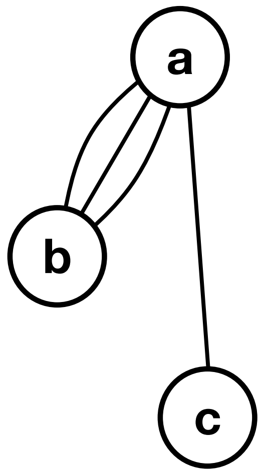
Simple Graph

- Loops, multi edges, and simple graphs
  - An edge of the form (v, v) is said to be a **self-loop**
  - A graph permitted to have multiple edges (or parallel edges) between two vertices is called a **multigraph**
  - A graph is said to be **simple** if it contains no self-loops or multiedges

Simple graph



Multigraph



Graph Connectivity

- Paths
  - A **path** is a sequence of edges connecting two vertices
  - A **simple path** never goes through any vertex more than once

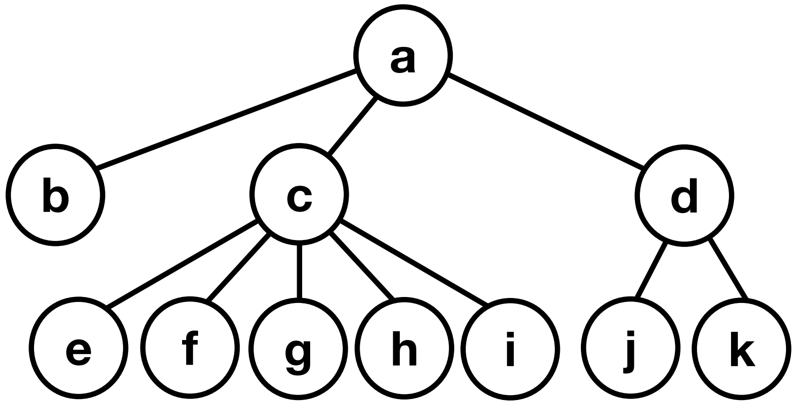
- Connectivity
  - A graph is **connected** if there is a path between any two vertices
  - Any subgraph that is connected can be referred to as a **connected component**
  - A directed graph is **strongly connected** if there is always a directed path between vertices

Trees and DAGs

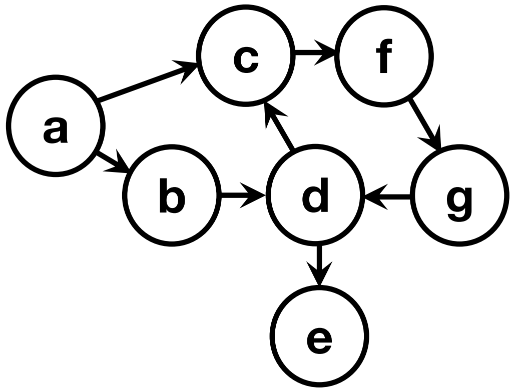
- A **cycle** is a path starting and ending at the same vertex. A cycle in which no vertex is repeated other than the starting vertex is said to be a **simple cycle**
- An undirected graph with no cycles is a **tree** if it is connected, or a **forest** otherwise
  - A **directed tree** is a directed graph which would be a tree if the directions on the edges were ignored
- A directed graph with no directed cycles is said to be a **directed acyclic graph** (**DAG**)

Examples

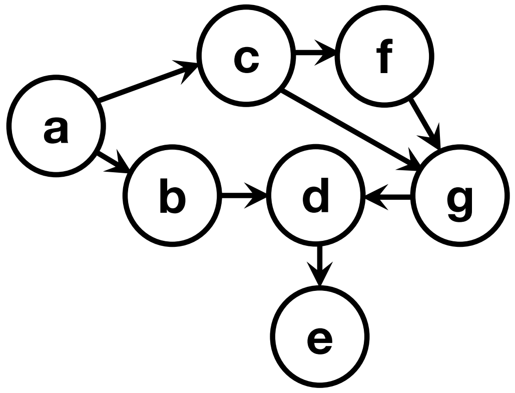
Tree



Directed graphs with cycles

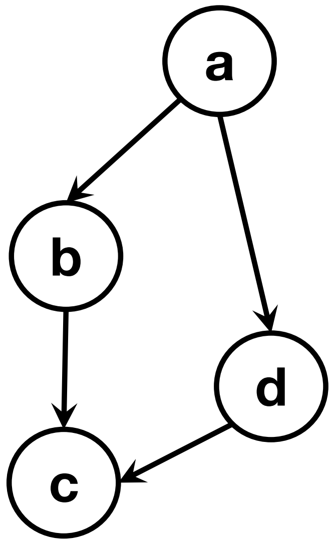


Directed acyclic graph (DAG)



Graph Traversal

- Purpose: visit all the vertices in a particular order, check/update their properties along the way
- Commonly used algorithms: Depth-first search (DFS); Breadth-first search (BFS)

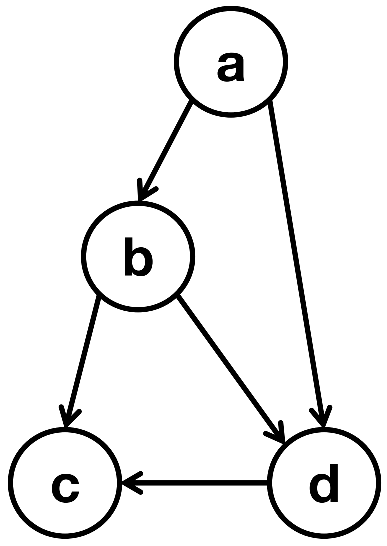


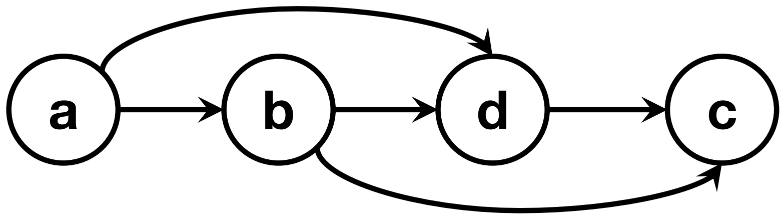
DFS order (from node a):
a → ?

BFS order:
a → ?

Topological Sort

- A **topological order of a directed graph** is an ordering of nodes where all edges go from an earlier vertex (left) to a later vertex (right)
  - Feasible if and only if the subject graph is a DAG





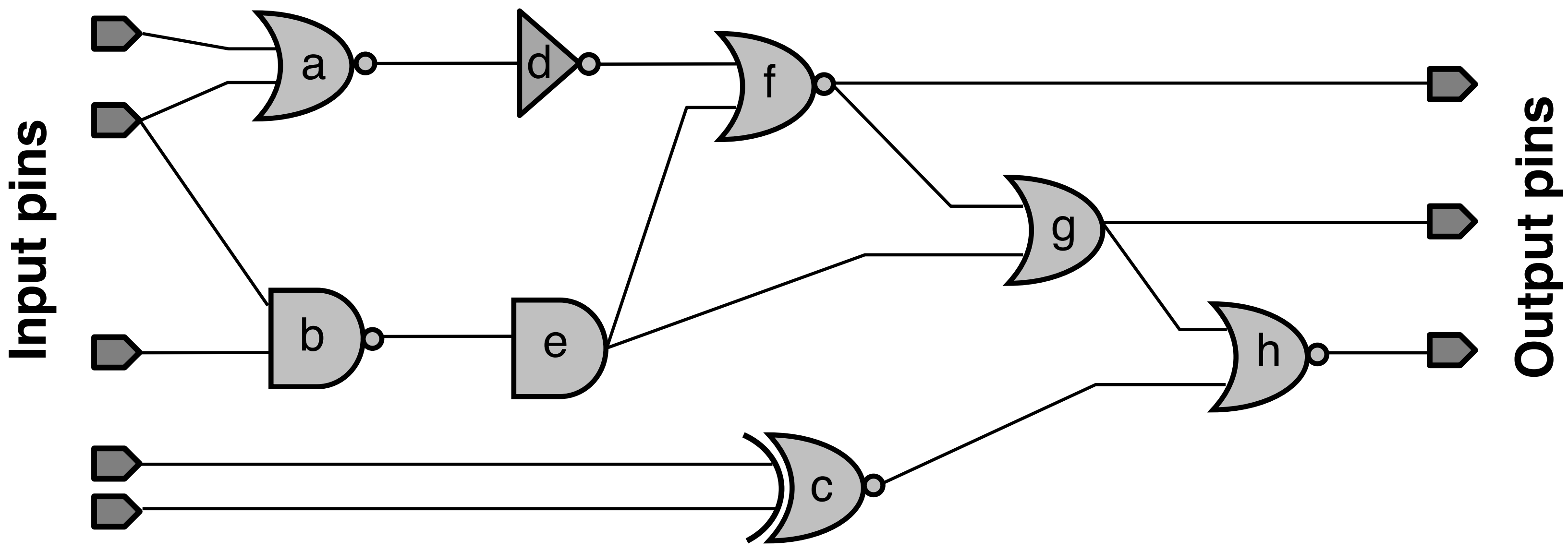
Application in EDA: Static Timing Analysis

- In circuit graphs, **static timing analysis** (STA) refers to the problem of finding the delays from the input pins of the circuit (esp. nodes) to each gate
  - In sequential circuits, flip-flop (FF) input acts as output pin, FF output acts as input pin
  - Max delay of the output pins determines clock period
  - **Critical path** is a path with max delay among all paths

- Two important terms
  - **Required time**: The time that the data signal needs to arrive at certain endpoint on a path to ensure the timing is met
  - **Arrival time**: The time that the data signal actually arrives at certain endpoint on a path

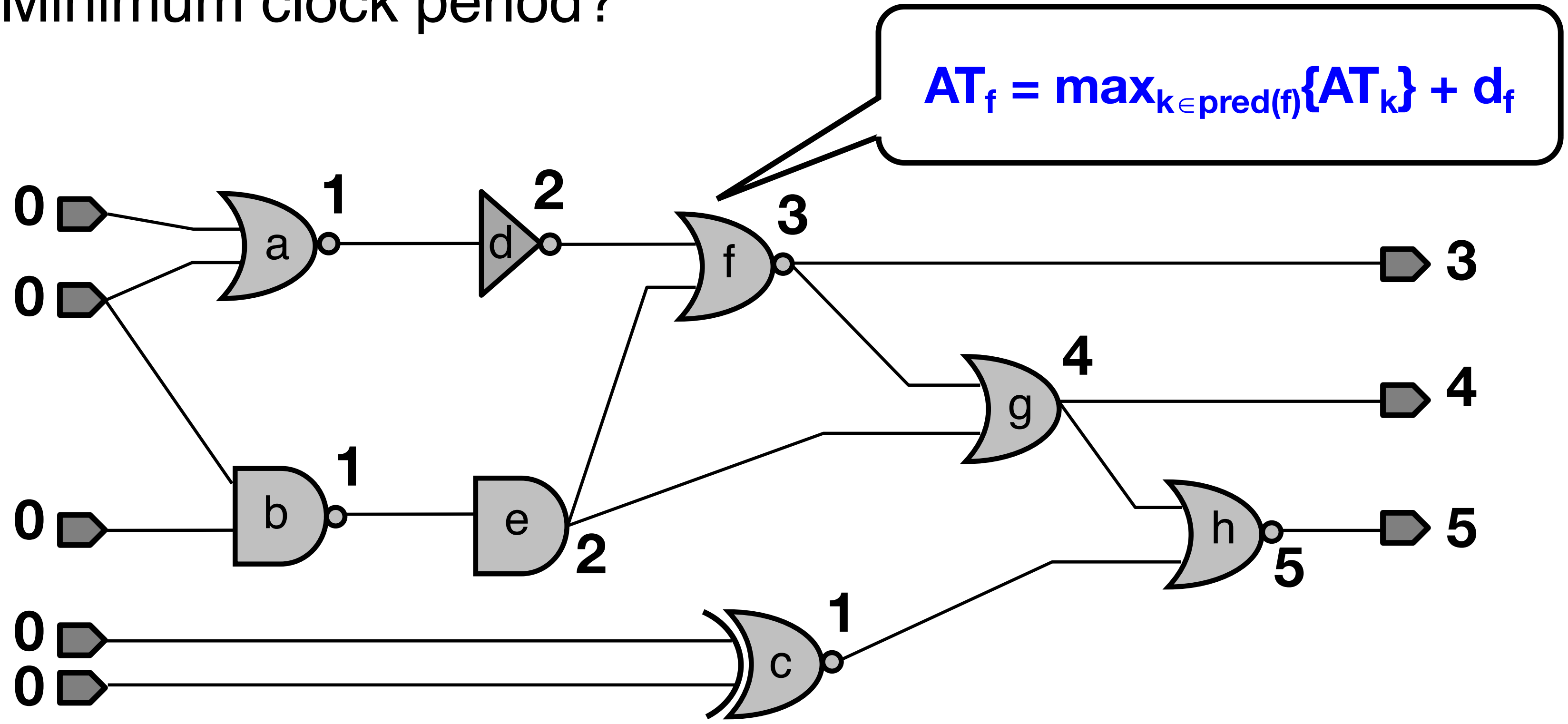
STA: An Example

- **pred**(n): predecessors of node n
  - e.g., **pred**(f) = {d, e}
- **succ**(n): successors of node n
  - e.g., **succ**(e) = {f, g}



STA: Arrival Times

- Assumptions
  - All inputs arrive at time 0
  - All gate delays = 1ns (d = 1); all wire delays = 0
- Questions: **Arrival time** (AT) of each gate output? Minimum clock period?

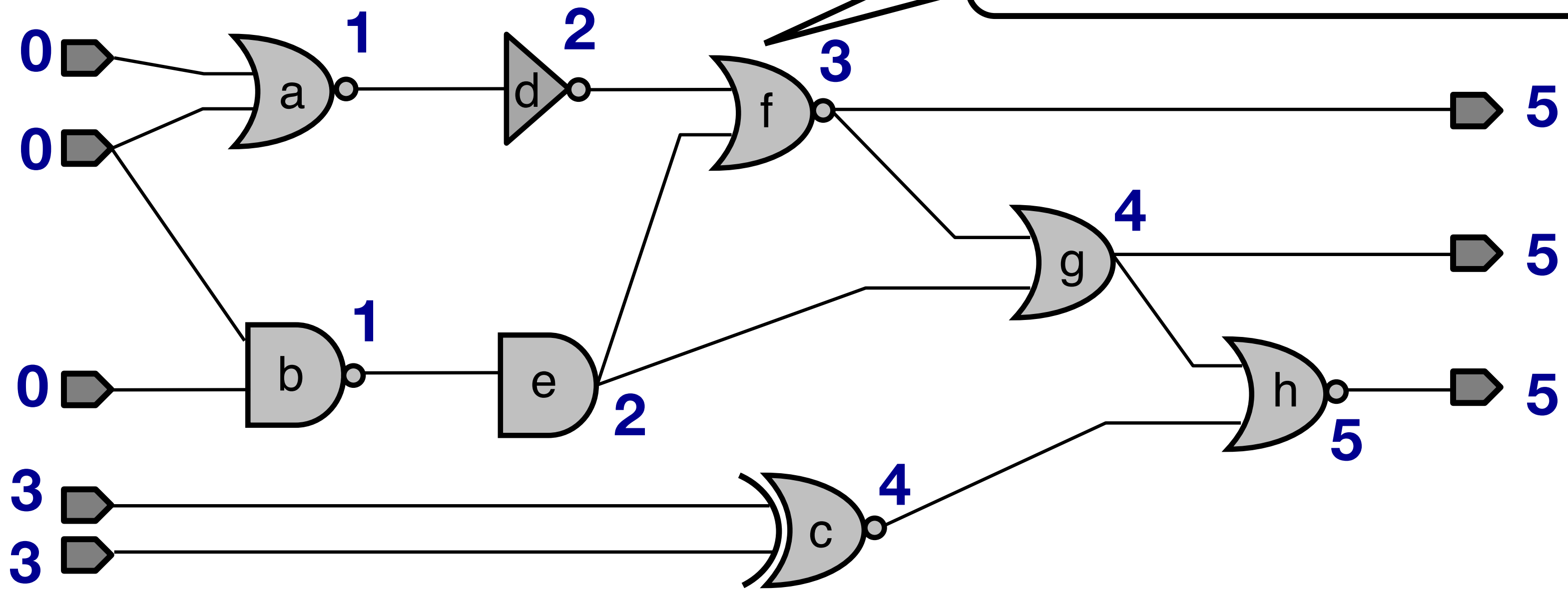


Gates are visited in a topological order

STA: Required Times

- Assumptions
  - All inputs arrive at time 0
  - All gate delays = 1ns (d = 1); all wire delays = 0
  - Clock period = 5ns (200MHz frequency)
- Question: **Required time** (RT) of each gate output in order to meet the clock period?

$\mathbf{RT_f = \min_{k \in succ(f)}\{RT_k - d_k\}}$



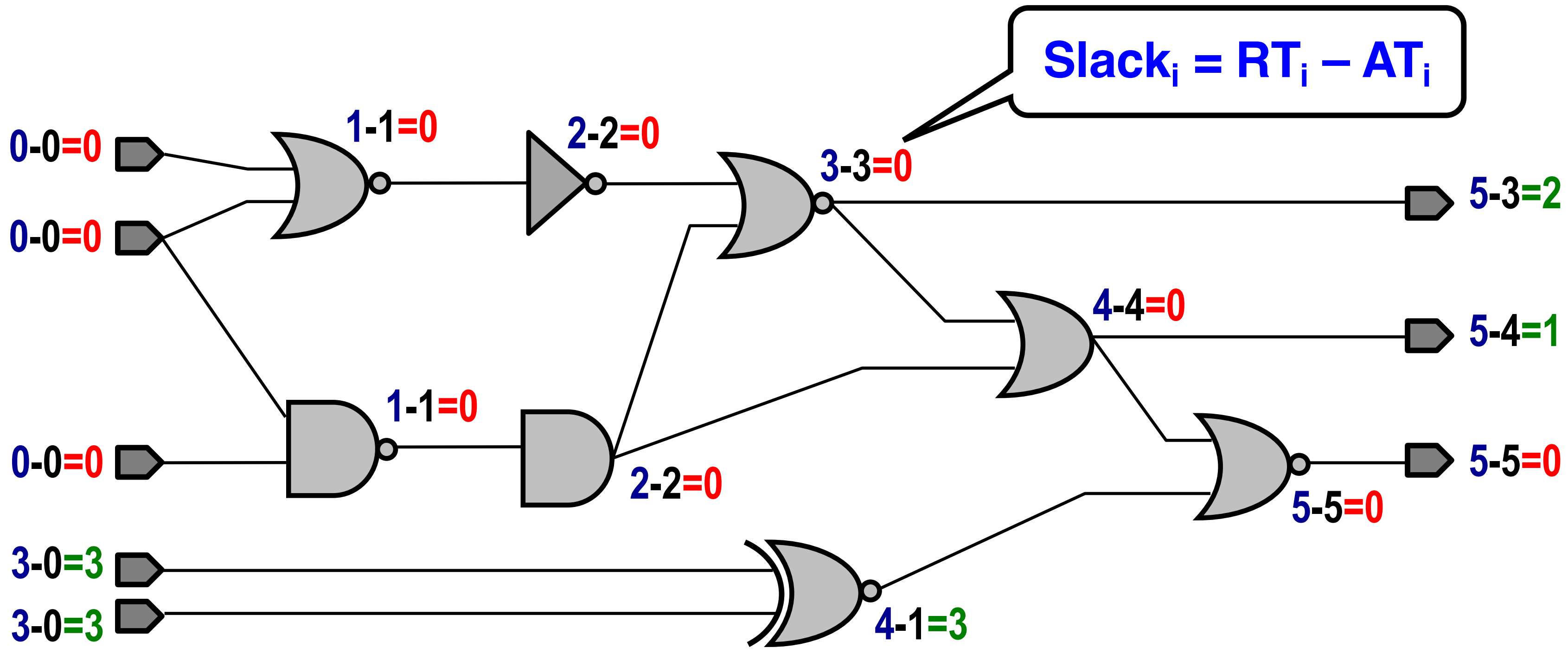
Gates are visited in a reverse topological order

STA: Slacks

- In addition to the arrival time and required time of each node, we are interested in knowing the **slack** (= RT - AT) of each node / edge
  - Negative slacks indicate unsatisfied timing constraints
  - Positive slacks often present opportunities for additional (area/power) optimization
  - Node on the **critical path** have zero slacks

STA: Use of Slacks

- Assumptions:
  - All inputs arrive at time 0
  - All gate delays = 1ns, wire delay = 0
  - Clock period = 5ns
- Question: What is the maximum slowdown of each gate without violating timing?



Next Lecture

- Binary decision diagrams (BDDs)

Acknowledgements

- These slides contain/adapt materials from / developed by
  - Prof. David Pan (UT Austin)
  - “VLSI Physical Design: From Graph Partitioning to Timing Closure” authored by Prof. Andrew B. Kahng, Prof. Jens Lienig, Prof. Igor L. Markov, Dr. Jin Hu