

ECE 6745 Complex Digital ASIC Design

Topic 13: Physical Design Automation Algorithms

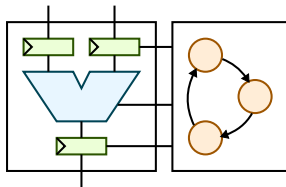
Christopher Batten

School of Electrical and Computer Engineering
Cornell University

<http://www.csl.cornell.edu/courses/ece6745>

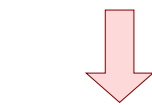
Part 3: CAD Algorithms

Topic 12 Synthesis Algorithms



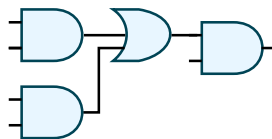
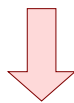
$$x = a'bc + a'bc'$$

$$y = b'c' + ab' + ac$$



$$x = a'b$$

$$y = b'c' + ac$$



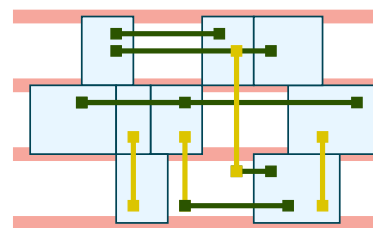
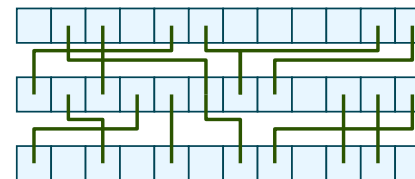
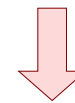
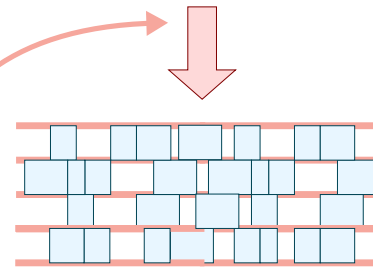
*RTL to Logic
Synthesis*

*Technology
Independent
Synthesis*

*Technology
Dependent
Synthesis*

Topic 13 Physical Design Automation Algorithms

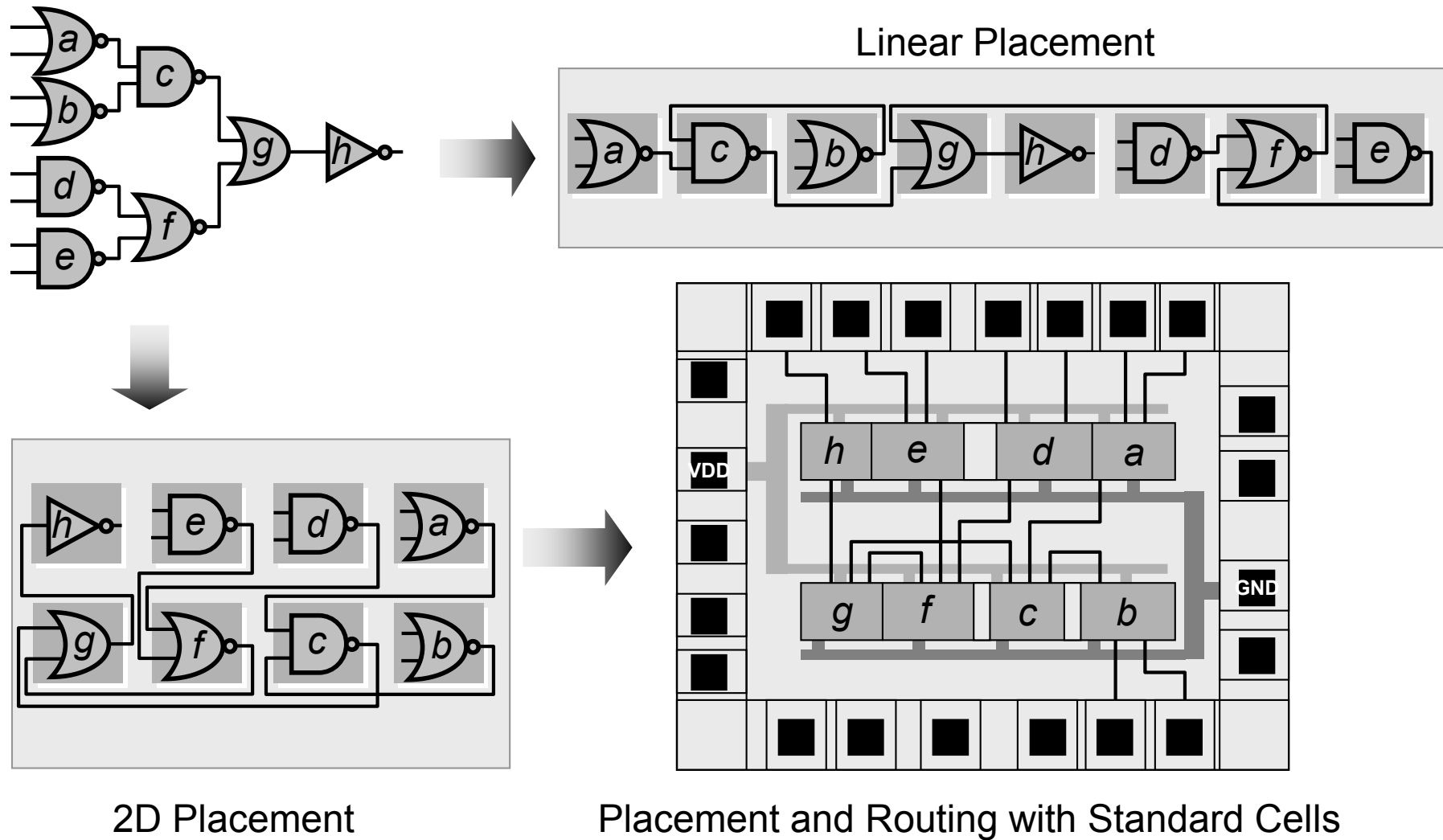
Placement



*Global
Routing*

*Detailed
Routing*

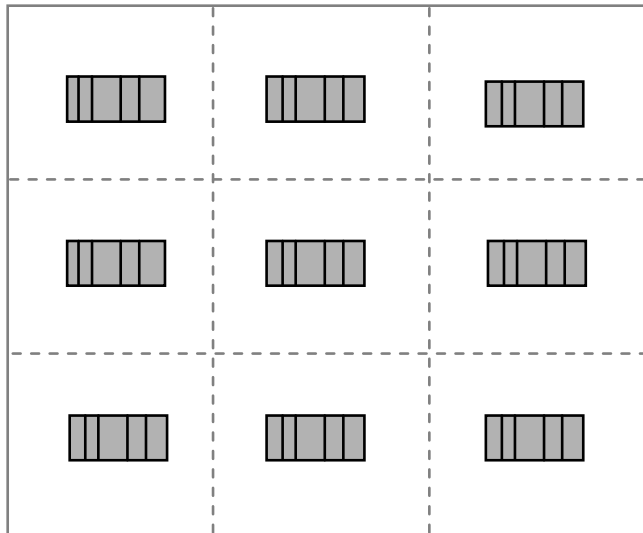
Placement



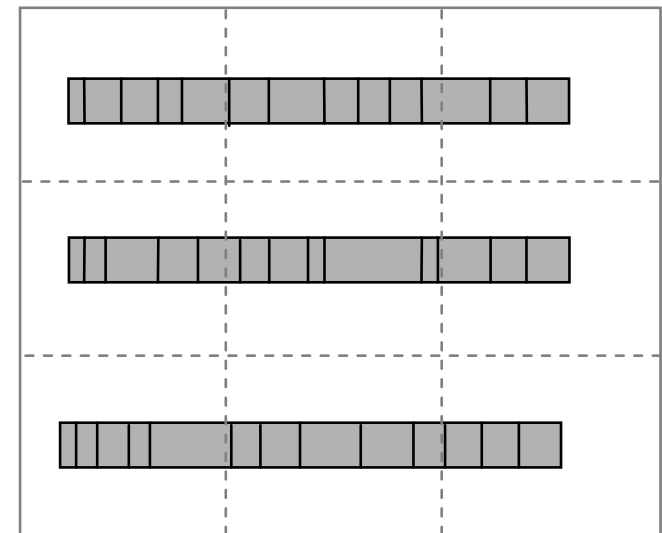
Adapted from [Khang'11]

Global vs. Detailed Placement

Global
Placement



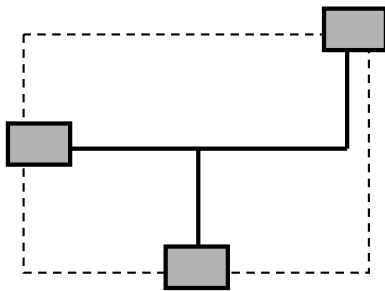
Detailed
Placement



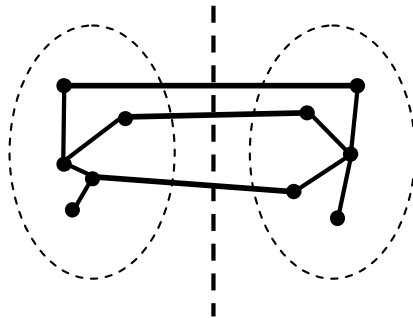
Adapted from [Khang'11]

Placement Objective Functions

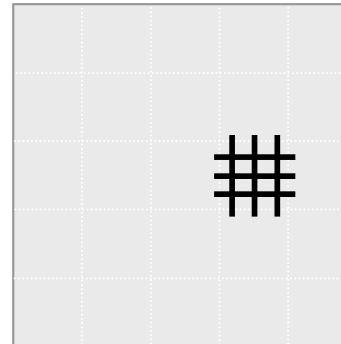
Total
Wirelength



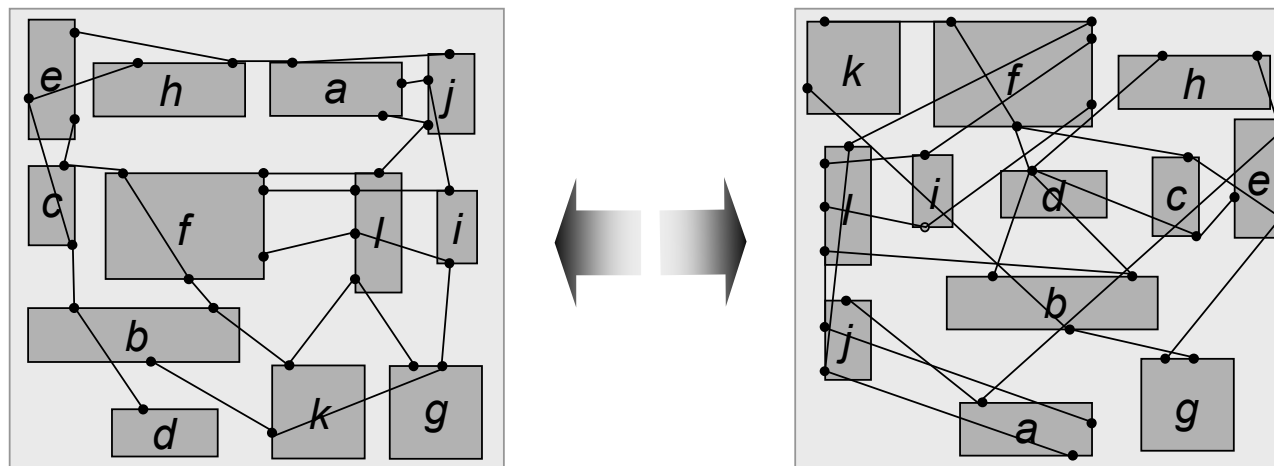
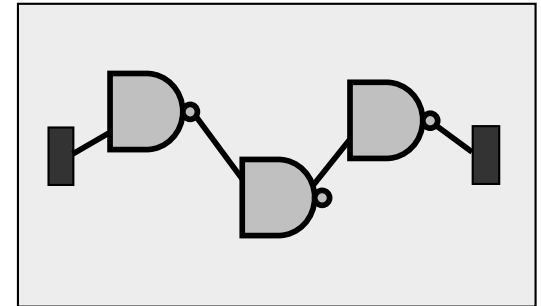
Number of
Cut Nets



Wire
Congestion

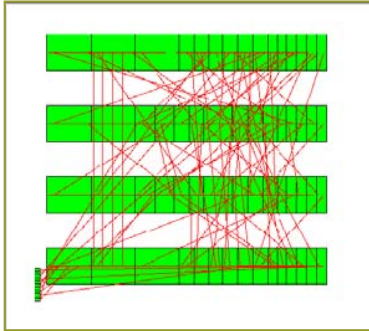


Signal
Delay



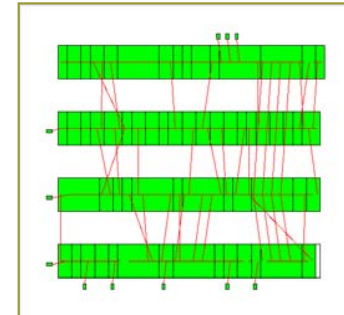
Adapted from [Khang'11]

Good vs. Bad Placement



Bad placement causes routing congestion resulting in:

- Increases in circuit area (cost) and wiring
- Longer wires → more capacitance
 - Longer delay
 - Higher dynamic power dissipation



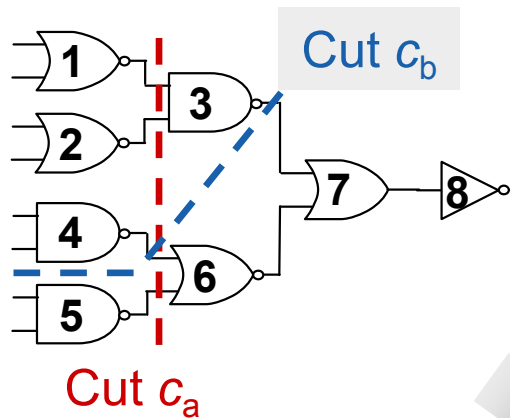
Good placement

- Circuit area (cost) and wiring decreases
- Shorter wires → less capacitance
 - Shorter delay
 - Less dynamic power dissipation

Adapted from [Devadas'06]

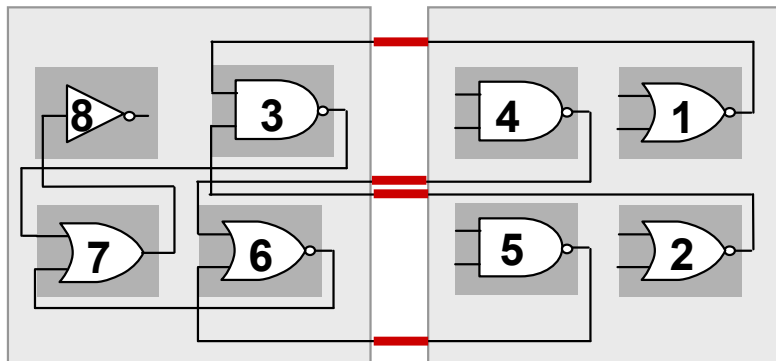
Partitioning

Circuit:



Block A

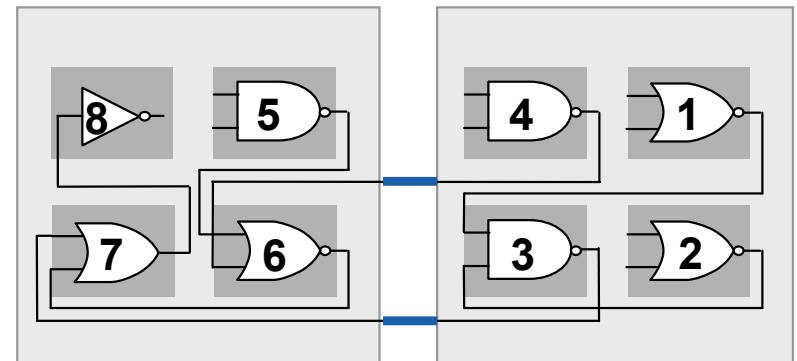
Block B



Cut c_a : four external connections

Block A

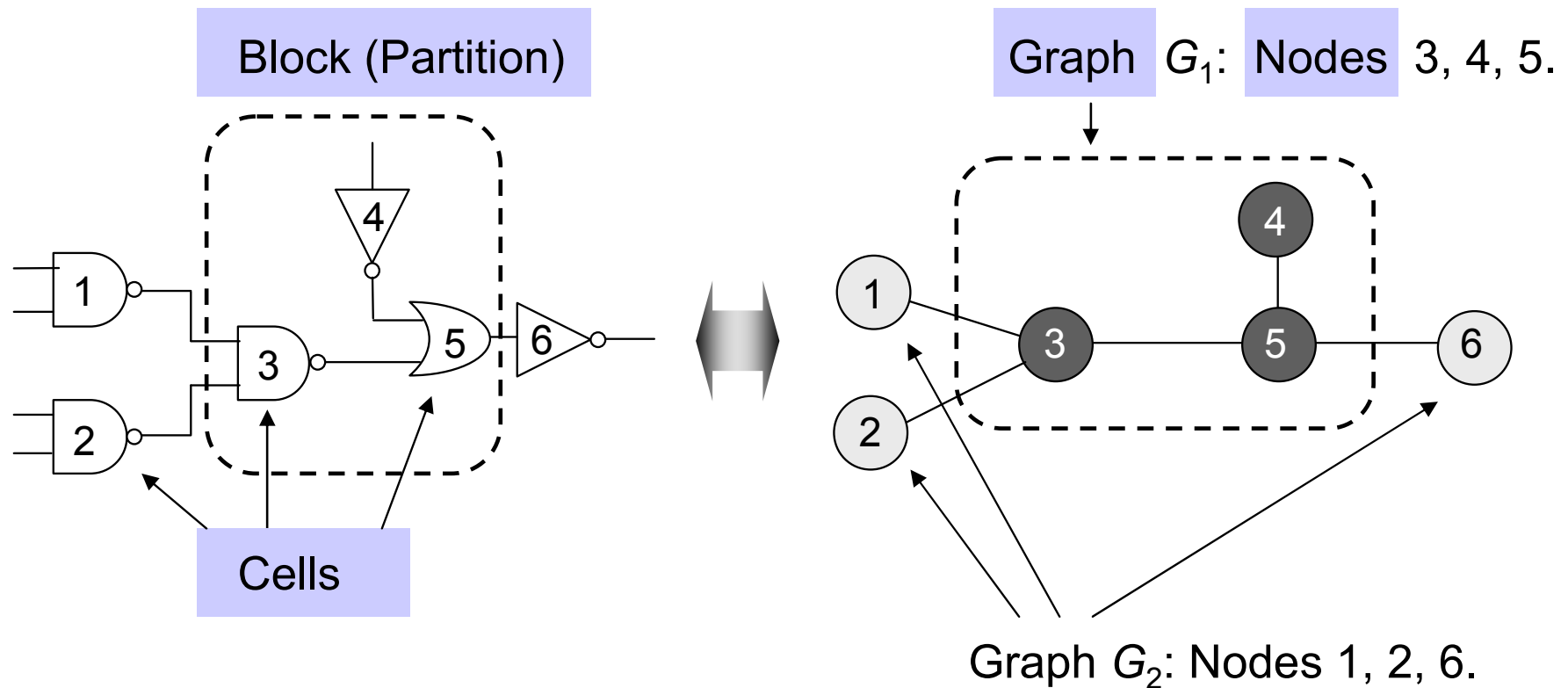
Block B



Cut c_b : two external connections

Adapted from [Khang'11]

Partitioning Terminology



Collection of cut edges

Cut set: (1,3), (2,3), (5,6),

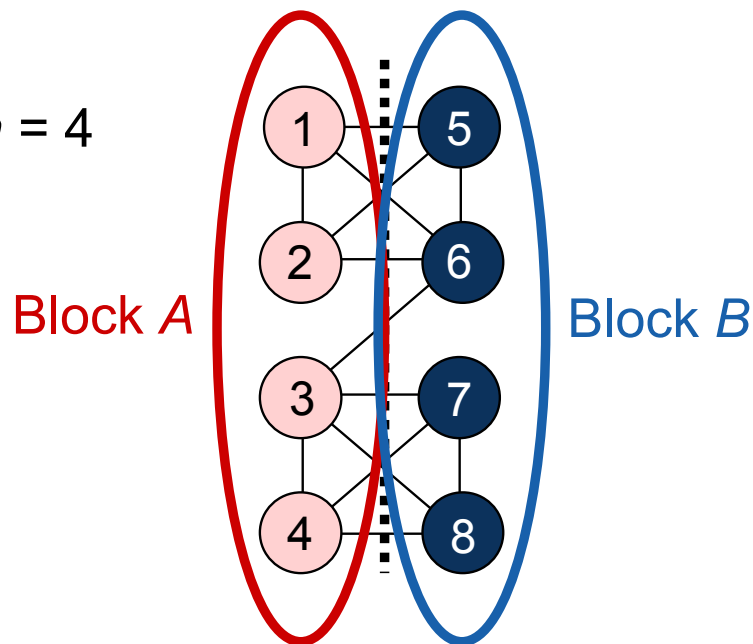
Adapted from [Khang'11]

Kernighan-Lin (KL) Algorithm

Given: A graph with $2n$ nodes where each node has the same weight.

Goal: A partition (division) of the graph into two disjoint subsets A and B with minimum cut cost and $|A| = |B| = n$.

Example: $n = 4$



Adapted from [Khang'11]

Kernighan-Lin (KL) Algorithm Concepts

Cost $D(v)$ of moving a node v

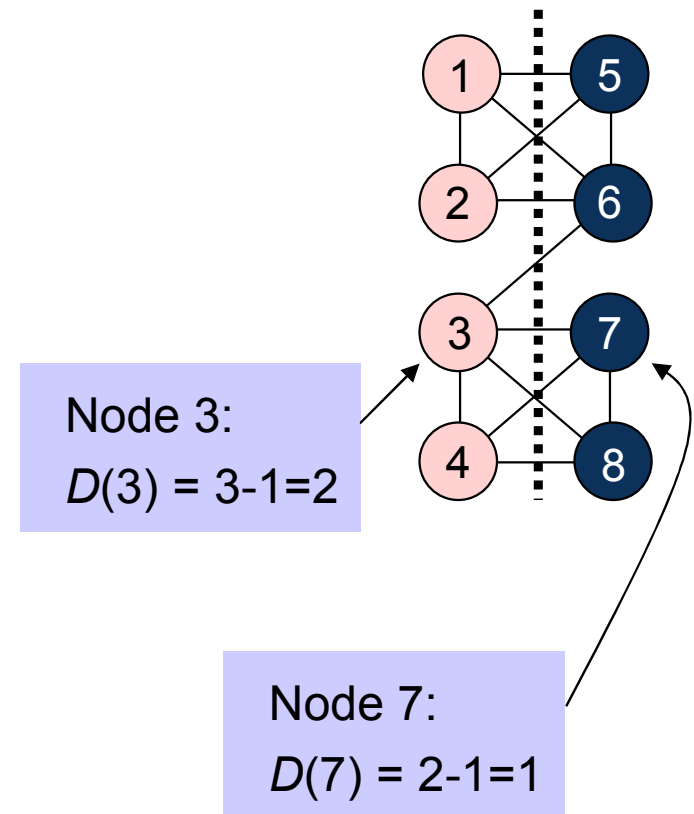
$$D(v) = |E_c(v)| - |E_{nc}(v)| ,$$

where

$E_c(v)$ is the set of v 's incident edges that are cut by the cut line, and

$E_{nc}(v)$ is the set of v 's incident edges that are not cut by the cut line.

High costs ($D > 0$) indicate that the node should move, while low costs ($D < 0$) indicate that the node should stay within the same partition.



Adapted from [Khang'11]

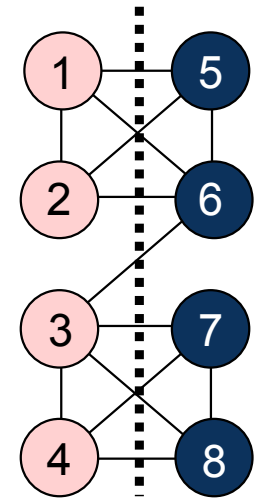
Kernighan-Lin (KL) Algorithm Concepts

Gain of swapping a pair of nodes a and b

$$\Delta g = D(a) + D(b) - 2 * c(a,b),$$

where

- $D(a)$, $D(b)$ are the respective costs of nodes a , b
- $c(a,b)$ is the connection weight between a and b :
If an edge exists between a and b ,
then $c(a,b)$ = edge weight (here 1),
otherwise, $c(a,b) = 0$.



The gain Δg indicates how useful the swap between two nodes will be

The larger Δg , the more the total cut cost will be reduced

Adapted from [Khang'11]

Kernighan-Lin (KL) Algorithm Concepts

Gain of swapping a pair of nodes a and b

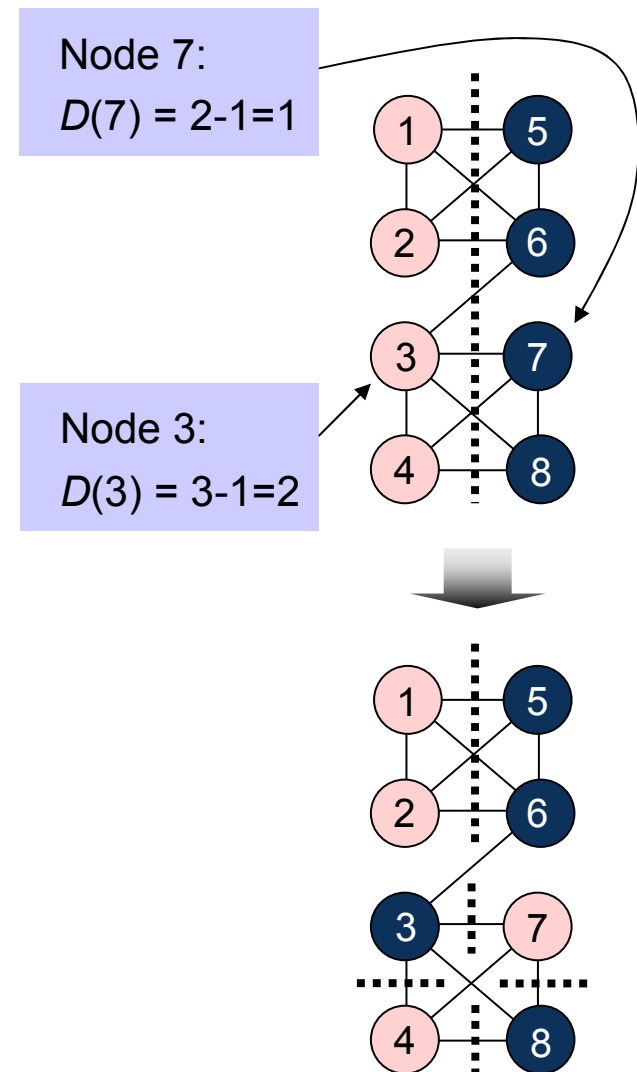
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otherwise, $c(a,b)$ = 0.

$$\Delta g(3,7) = D(3) + D(7) - 2 * c(a,b) = 2 + 1 - 2 = 1$$

=> Swapping nodes 3 and 7 would reduce the cut size by 1



Adapted from [Khang'11]

Kernighan-Lin (KL) Algorithm Concepts

Gain of swapping a pair of nodes a and b

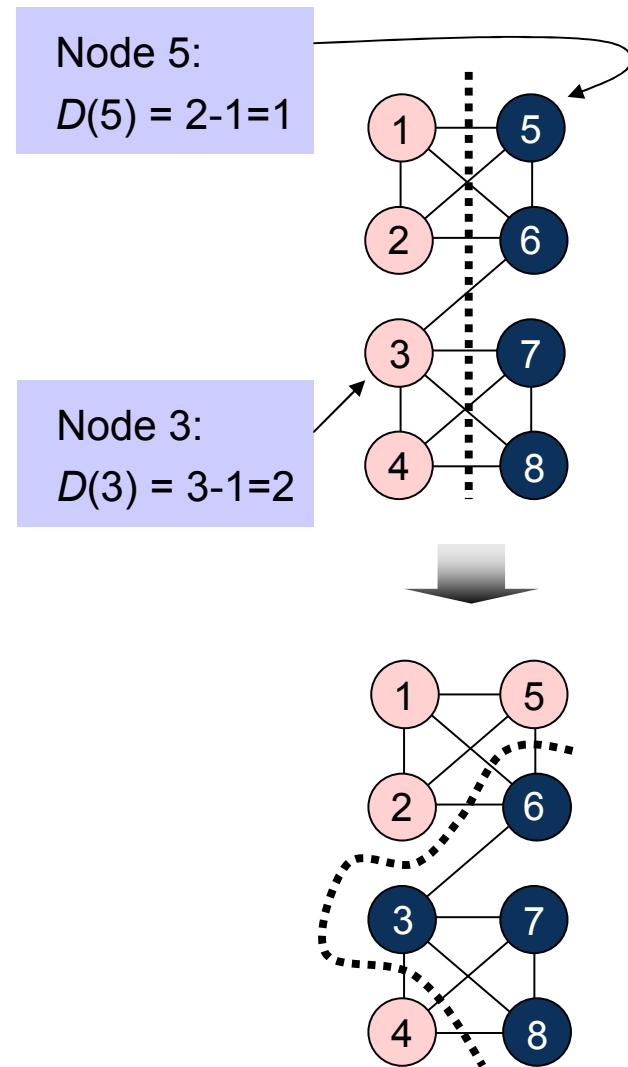
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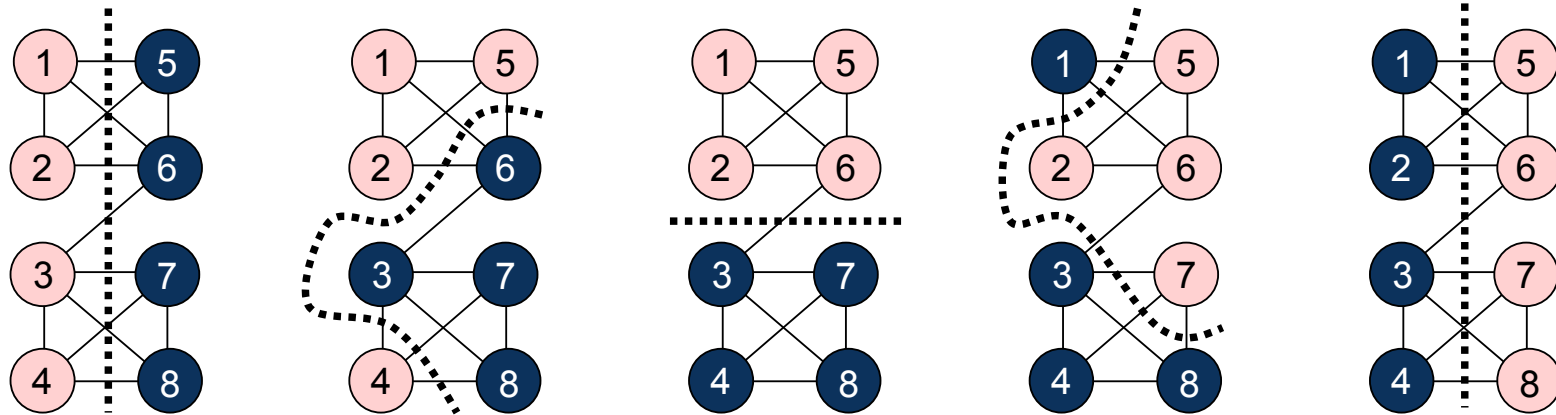
$$\Delta g(3,5) = D(3) + D(5) - 2 * c(a,b) = 2 + 1 - 0 = 3$$

=> Swapping nodes 3 and 5 would reduce the cut size by 3



Adapted from [Khang'11]

Kernighan-Lin (KL) Algorithm Example



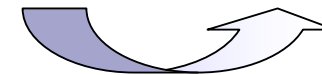
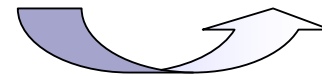
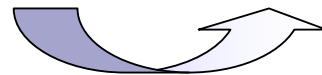
Cut cost: 9
Not fixed:
1,2,3,4,5,6,7,8

Cut cost: 6
Not fixed:
1,2,4,6,7,8

Cut cost: 1
Not fixed:
1,2,7,8

Cut cost: 7
Not fixed:
2,8

Cut cost: 9
Not fixed:
—



$D(1) = 1$ $D(5) = 1$
 $D(2) = 1$ $D(6) = 2$
 $D(3) = 2$ $D(7) = 1$
 $D(4) = 1$ $D(8) = 1$

$$\Delta g_1 = 2 + 1 - 0 = 3$$

Swap (3,5)

$$G_1 = \Delta g_1 = 3$$

$D(1) = -1$ $D(6) = 2$
 $D(2) = -1$ $D(7) = -1$
 $D(4) = 3$ $D(8) = -1$

$$\Delta g_2 = 3 + 2 - 0 = 5$$

Swap (4,6)

$$G_2 = G_1 + \Delta g_2 = 8$$

$D(1) = -3$ $D(7) = -3$
 $D(2) = -3$ $D(8) = -3$

$$\Delta g_3 = -3 - 3 - 0 = -6$$

Swap (1,7)

$$G_3 = G_2 + \Delta g_3 = 2$$

$D(2) = -1$ $D(8) = -1$

$$\Delta g_4 = -1 - 1 - 0 = -2$$

Swap (2,8)

$$G_4 = G_3 + \Delta g_4 = 0$$

Adapted from [Khang'11]

Kernighan-Lin (KL) Algorithm Example

$D(1) = 1$ $D(5) = 1$
 $D(2) = 1$ $D(6) = 2$
 $D(3) = 2$ $D(7) = 1$
 $D(4) = 1$ $D(8) = 1$

$$\Delta g_1 = 2 + 1 - 0 = 3$$

Swap (3,5)

$$G_1 = \Delta g_1 = 3$$

$D(1) = -1$ $D(6) = 2$
 $D(2) = -1$ $D(7) = -1$
 $D(4) = 3$ $D(8) = -1$

$$\Delta g_2 = 3 + 2 - 0 = 5$$

Swap (4,6)

$$G_2 = G_1 + \Delta g_2 = 8$$

$D(1) = -3$ $D(7) = -3$
 $D(2) = -3$ $D(8) = -3$

$$\Delta g_3 = -3 - 3 - 0 = -6$$

Swap (1,7)

$$G_3 = G_2 + \Delta g_3 = 2$$

$D(2) = -1$ $D(8) = -1$

$$\Delta g_4 = -1 - 1 - 0 = -2$$

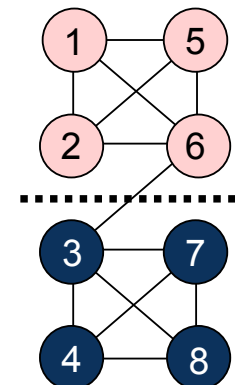
Swap (2,8)

$$G_4 = G_3 + \Delta g_4 = 0$$

Maximum positive gain $G_m = 8$ with $m = 2$.

Since $G_m > 0$, the first $m = 2$ swaps (3,5) and (4,6) are executed.

Since $G_m > 0$, more passes are needed until $G_m \leq 0$.



Adapted from [Khang'11]

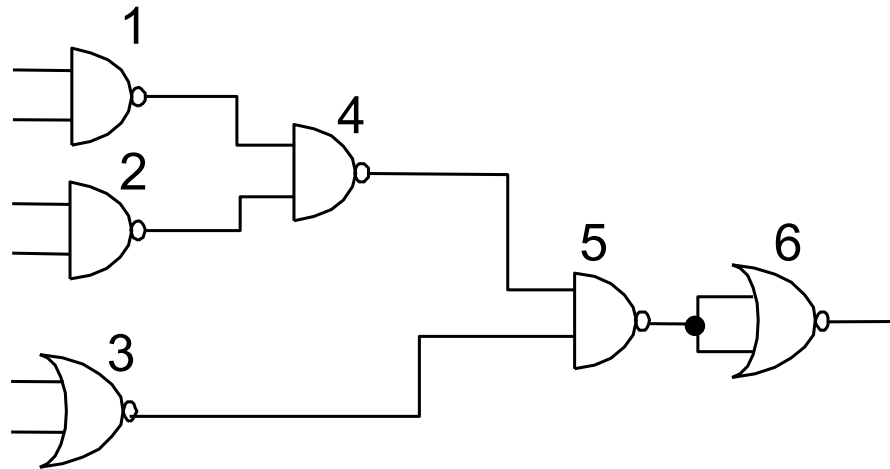
Min-Cut Placement

- ▶ Use partitioning algorithm (such as KL) to divide netlist into two regions
- ▶ Use partitioning algorithm to recursively divide the two partitions into two smaller regions (four regions total)
- ▶ Each cut heuristically minimizes the number of cut nets

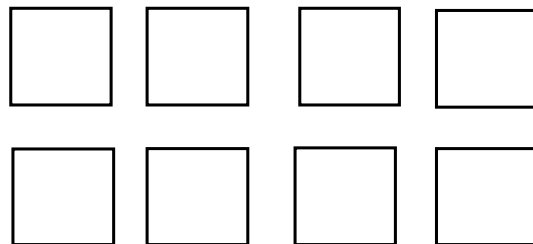
Adapted from [Khang'11]

Min-Cut Example

Given:

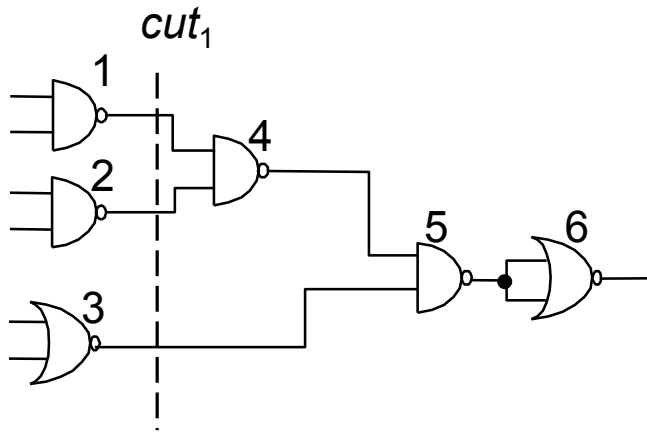


Task: 4 x 2 placement with minimum wirelength using alternative cutline directions and the KL algorithm

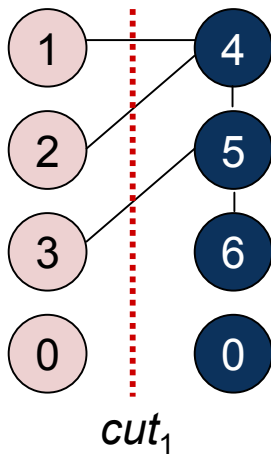


Adapted from [Khang'11]

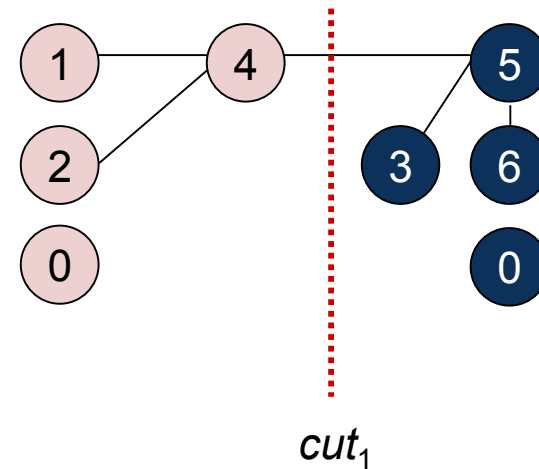
Min-Cut Example



Vertical cut cut_1 : $L=\{1,2,3\}$, $R=\{4,5,6\}$

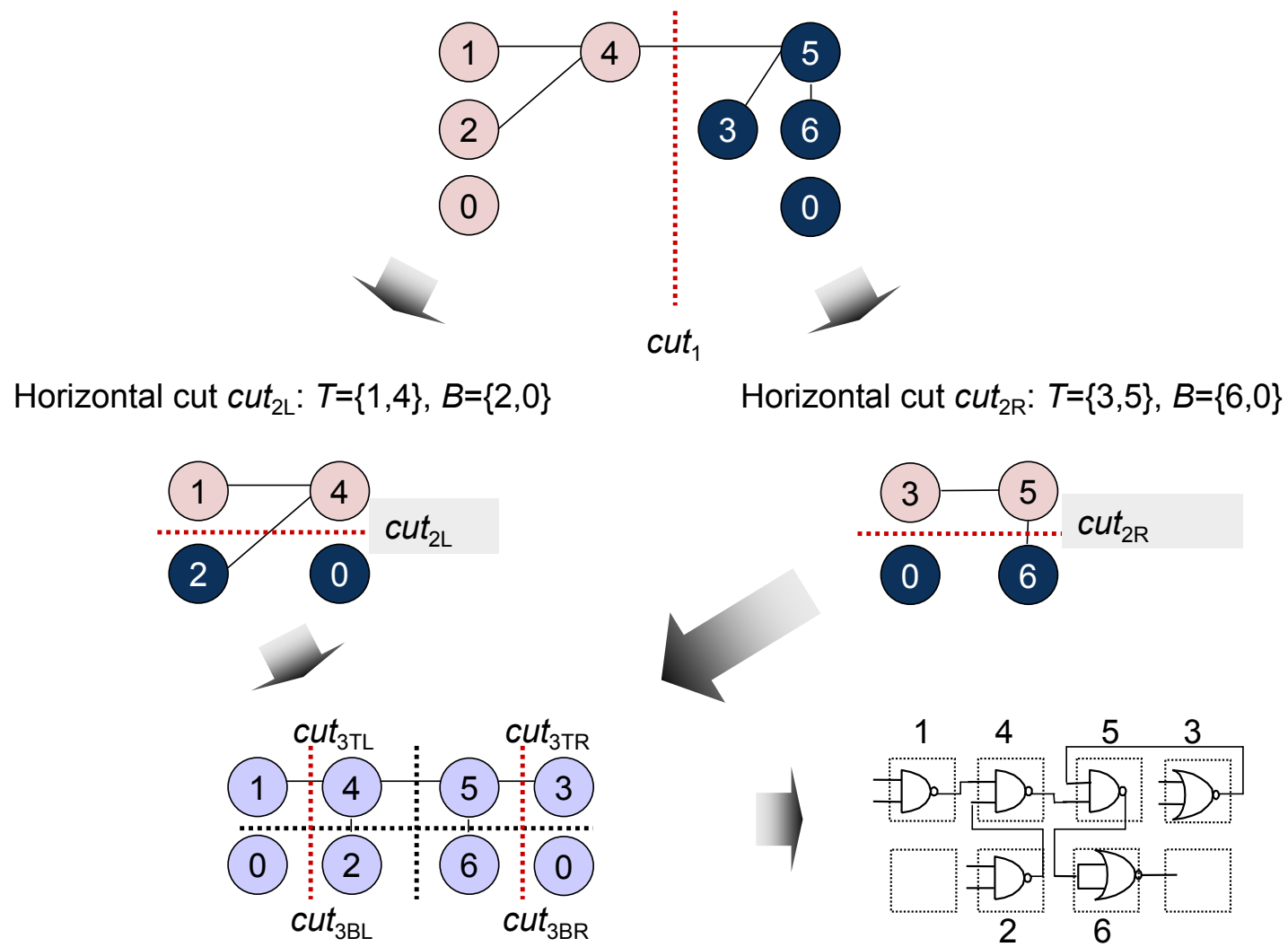


KL Algorithms



Adapted from [Khang'11]

Min-Cut Example



Adapted from [Khang'11]

Analytical Placement with Quadratic Placement Algorithm

- Objective function is quadratic; sum of (weighted) **squared Euclidean distance** represents placement objective function

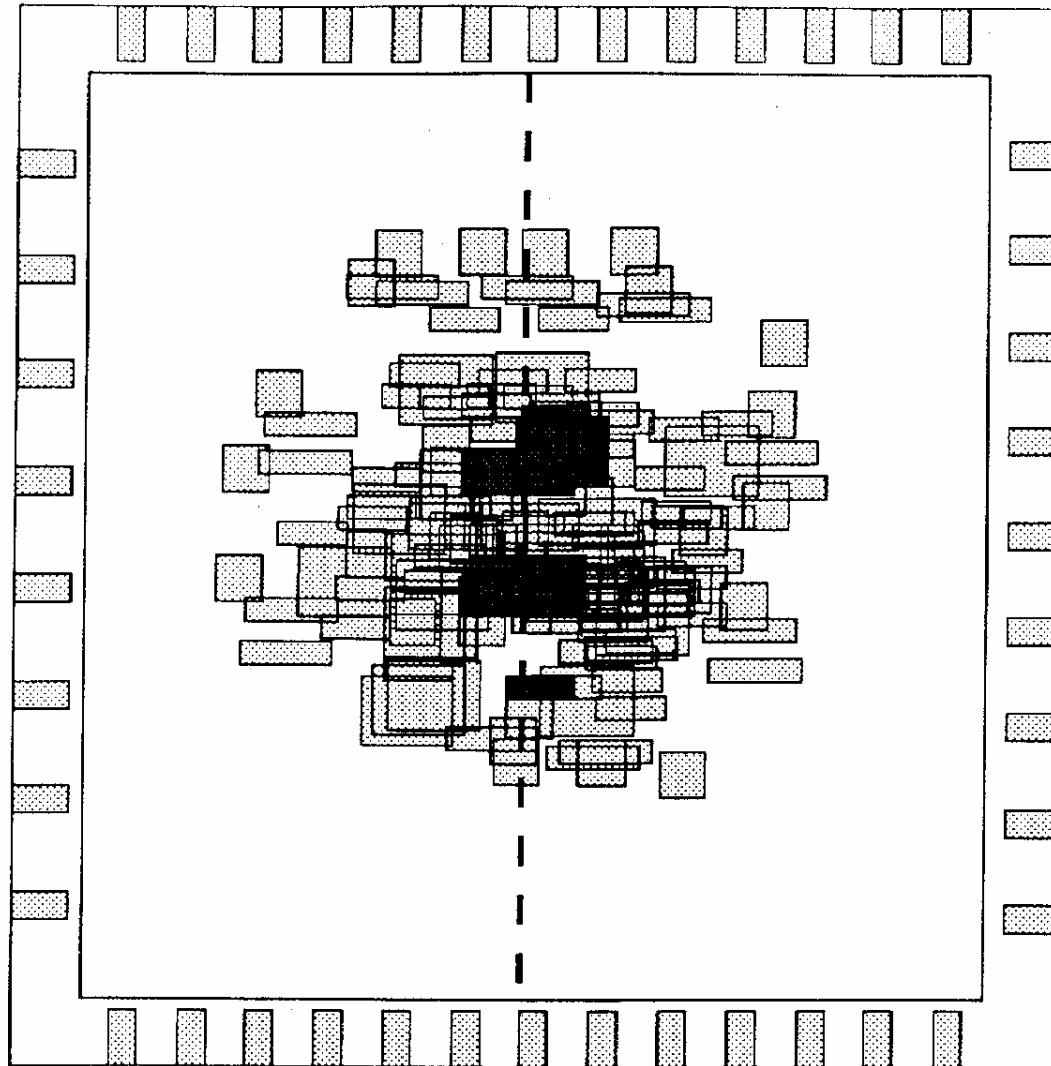
$$L(P) = \frac{1}{2} \sum_{i,j=1}^n c_{ij} \left((x_i - x_j)^2 + (y_i - y_j)^2 \right)$$

where n is the total number of cells, and $c(i,j)$ is the connection cost between cells i and j .

- Only two-point-connections
- Minimize objective function by equating its derivative to zero which reduces to solving a system of linear equations

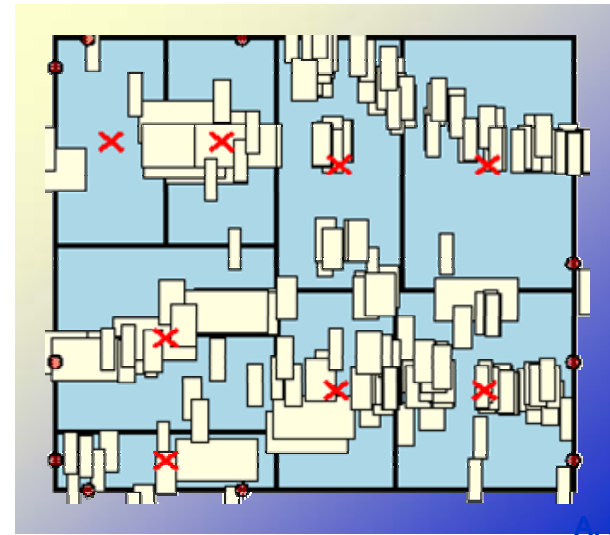
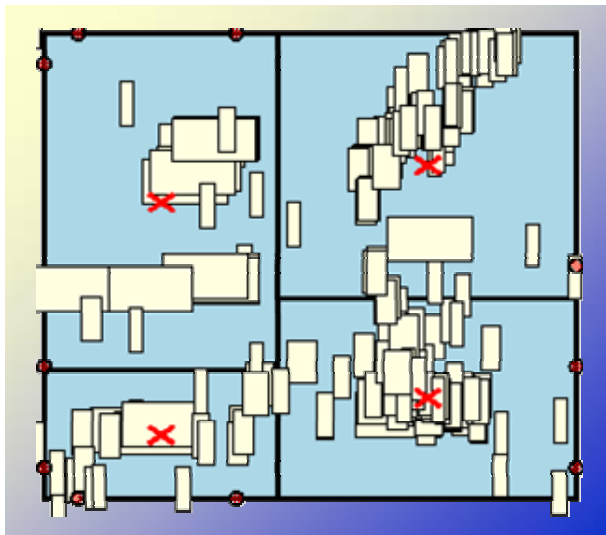
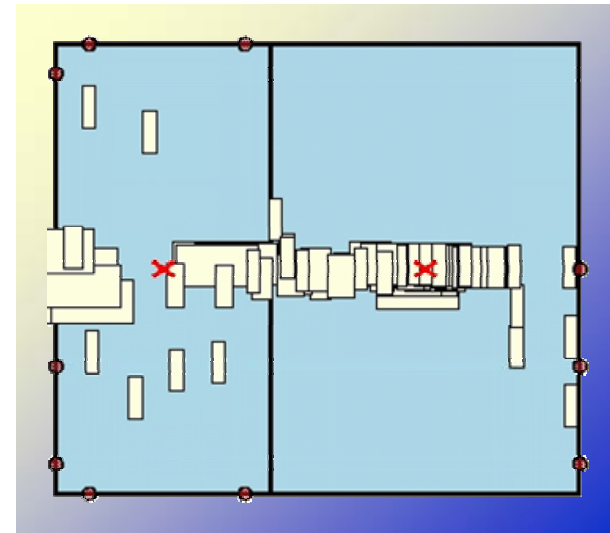
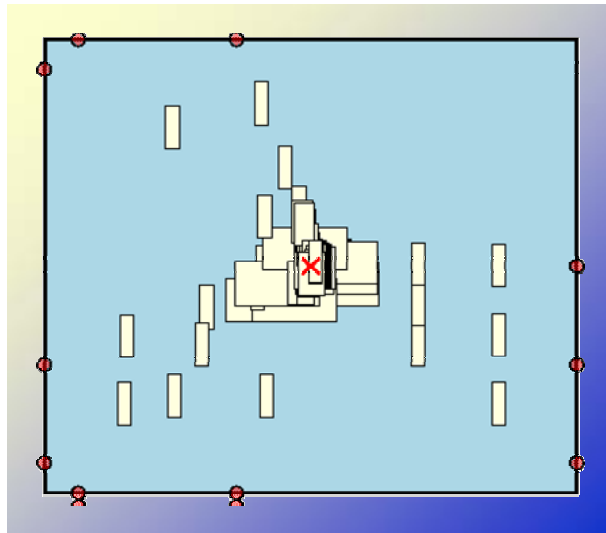
Adapted from [Khang'11]

Minimizing Objective Function Clumps Cells Together



Adapted from [Devadas'06]

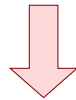
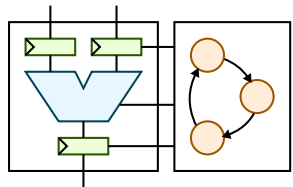
Using Partitioning to Pull Cells Apart



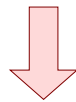
Adapted from [Devadas'06,Kahng'11]

Part 3: CAD Algorithms

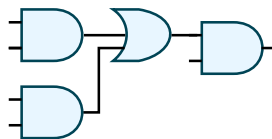
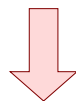
Topic 12 Synthesis Algorithms



$$\begin{aligned}x &= a'bc + a'bc' \\ y &= b'c' + ab' + ac\end{aligned}$$



$$\begin{aligned}x &= a'b \\ y &= b'c' + ac\end{aligned}$$



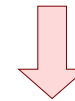
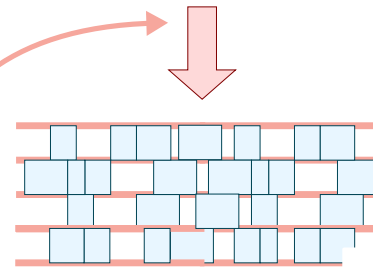
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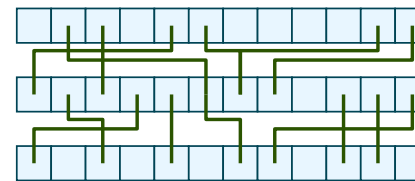
*Technology
Dependent
Synthesis*

Topic 13 Physical Design Automation

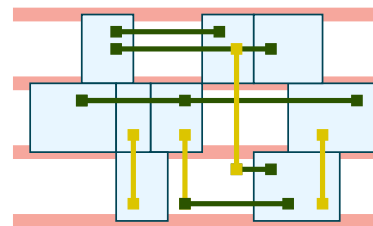
Placement



**Global
Routing**



*Detailed
Routing*



General Routing Problem

Netlist:

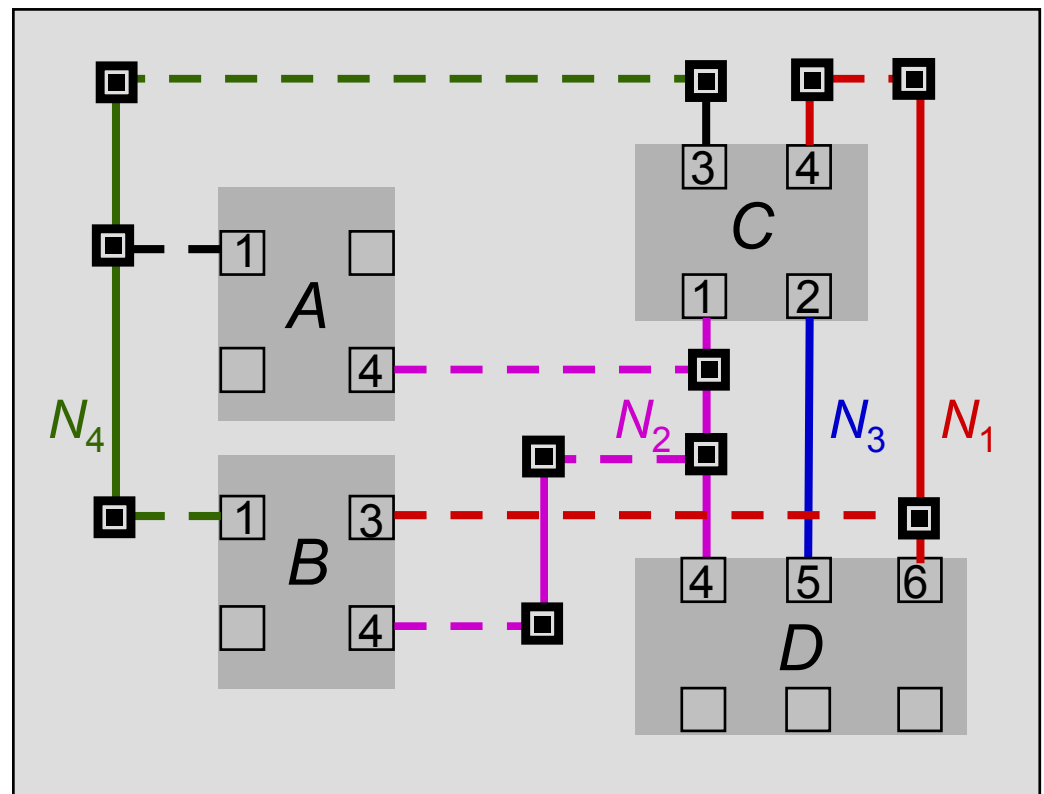
$$N_1 = \{C_4, D_6, B_3\}$$

$$N_2 = \{D_4, B_4, C_1, A_4\}$$

$$N_3 = \{C_2, D_5\}$$

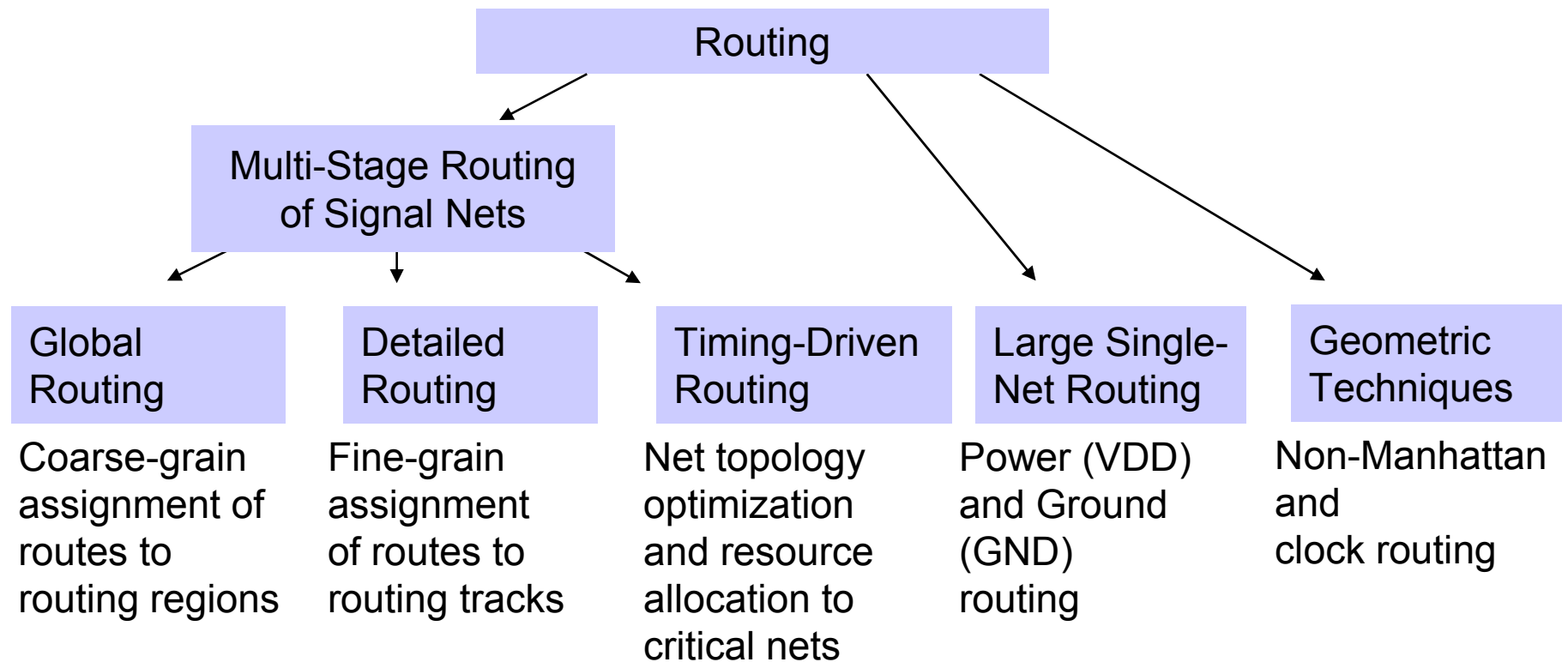
$$N_4 = \{B_1, A_1, C_3\}$$

Technology Information
(Design Rules)



Adapted from [Kahng'11]

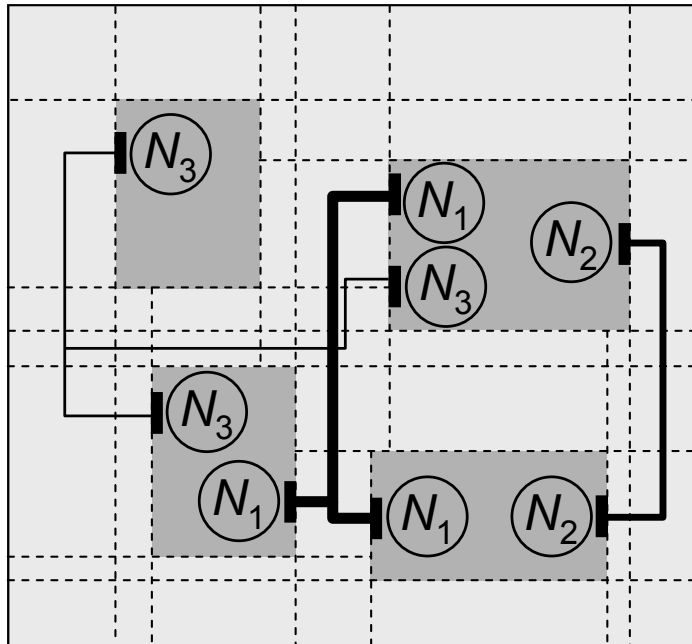
Routing Algorithm Taxonomy



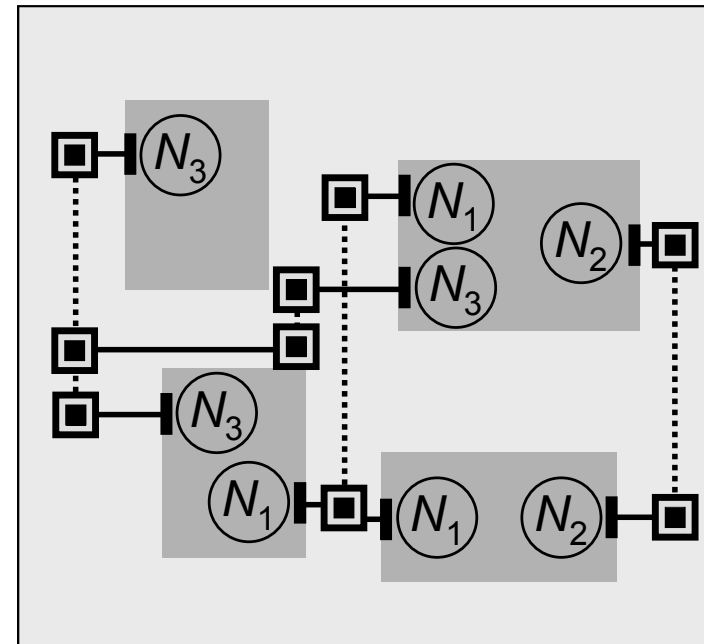
Adapted from [Kahng'11]

Global vs. Detailed Routing

Global Routing



Detailed Routing

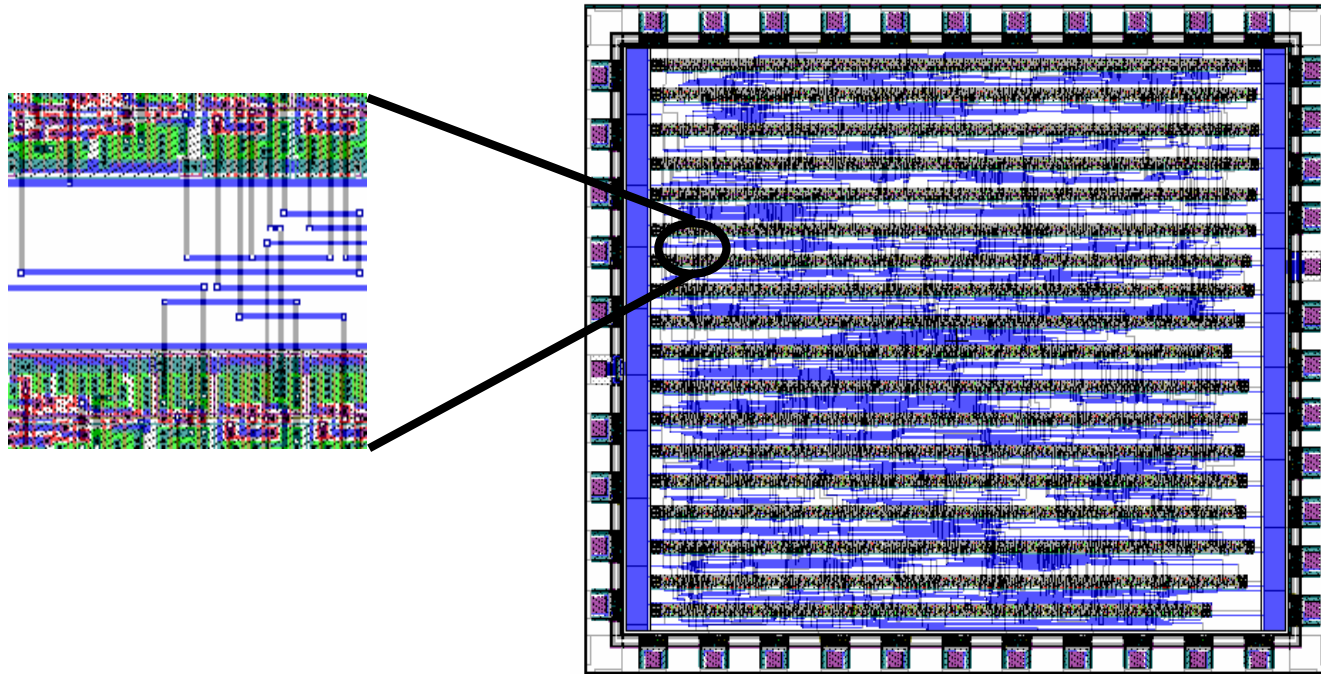


Adapted from [Kahng'11]

Terminology: Channel

Channel

Rectangular routing region with pins on two opposite sides



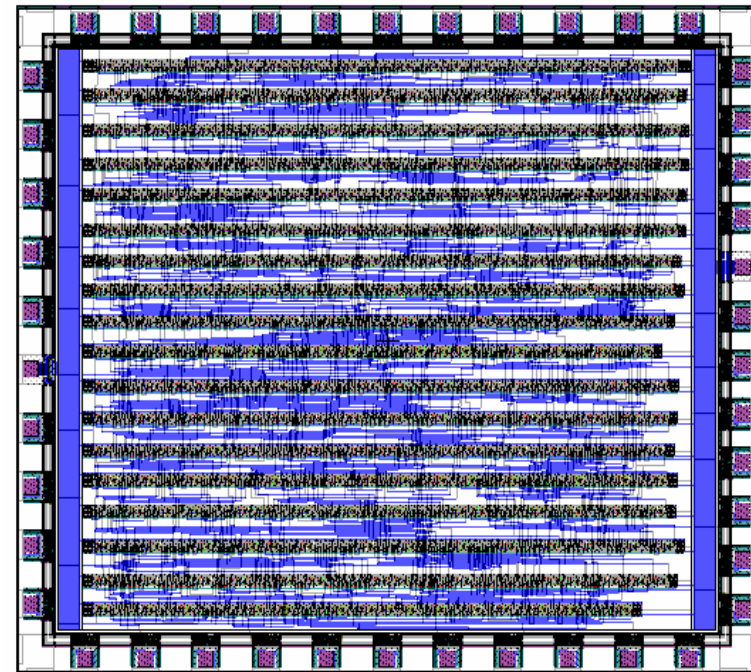
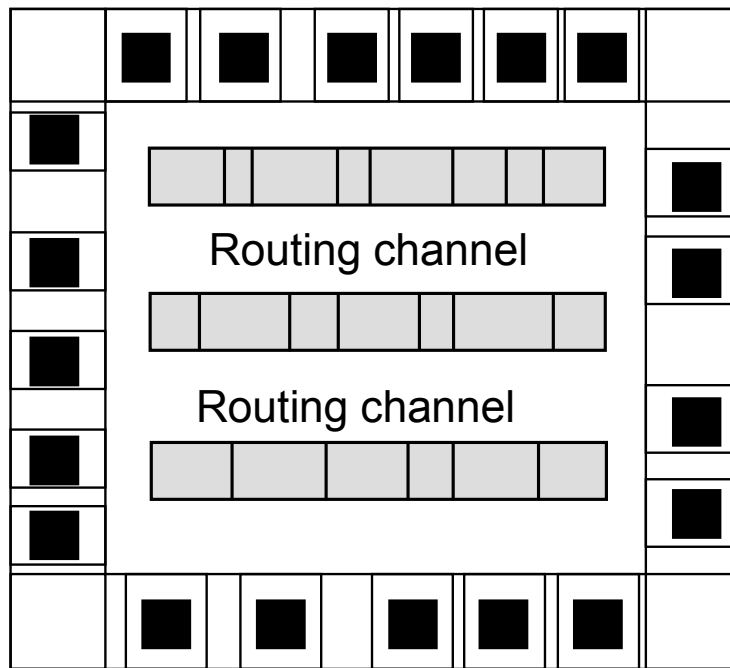
Standard cell layout (Two-layer routing)

Adapted from [Kahng'11]

Terminology: Channel

Channel

Rectangular routing region with pins on two opposite sides



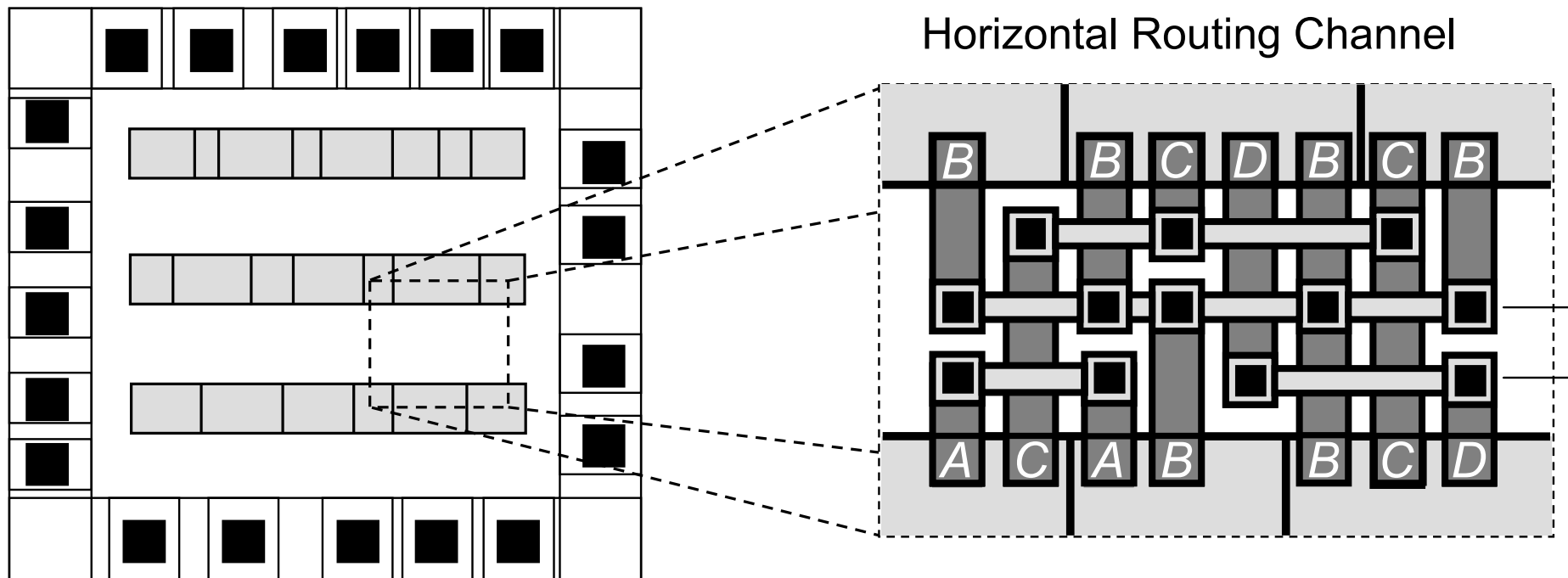
Standard cell layout (Two-layer routing)

Adapted from [Kahng'11]

Terminology: Capacity

Capacity

Number of available routing tracks or columns

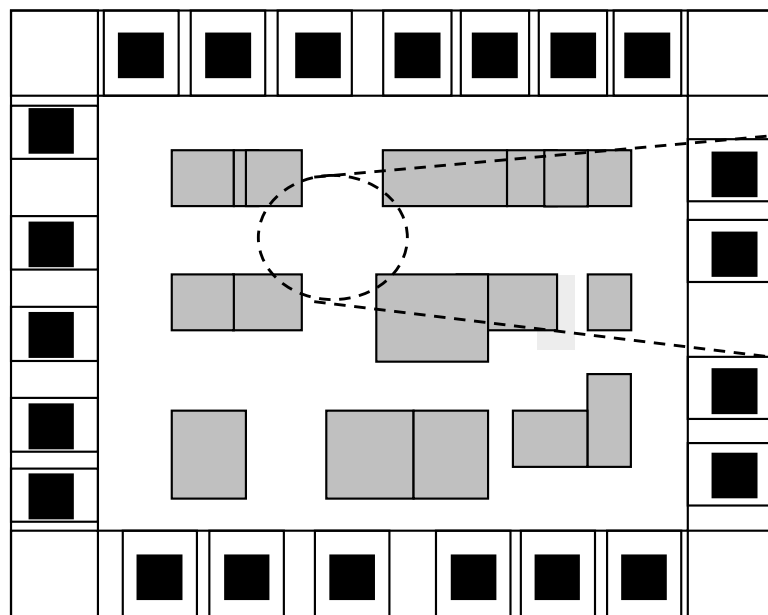


Adapted from [Kahng'11]

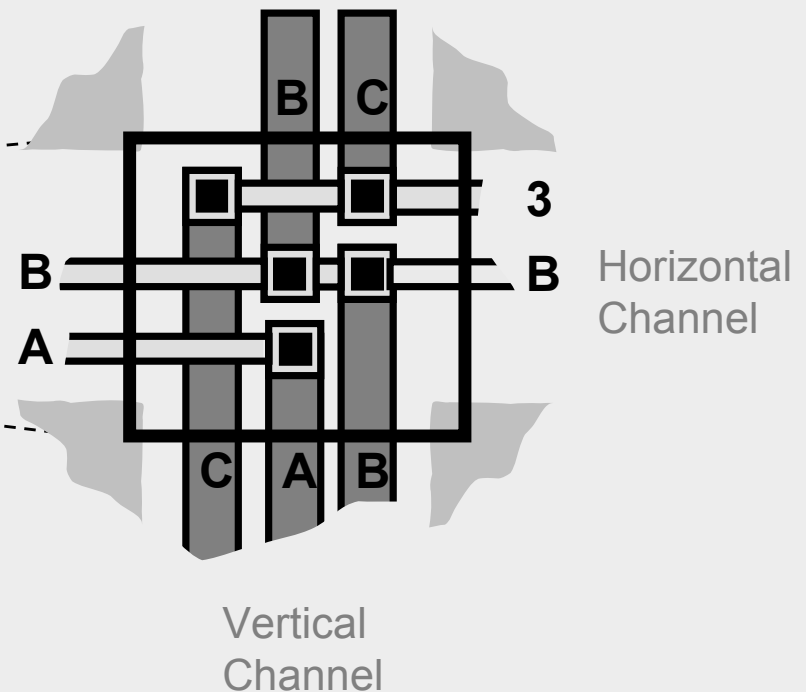
Terminology: Switchbox

Switchbox (Two-layer macro cell layout)

Intersection of horizontal and vertical channels



Horizontal Channel

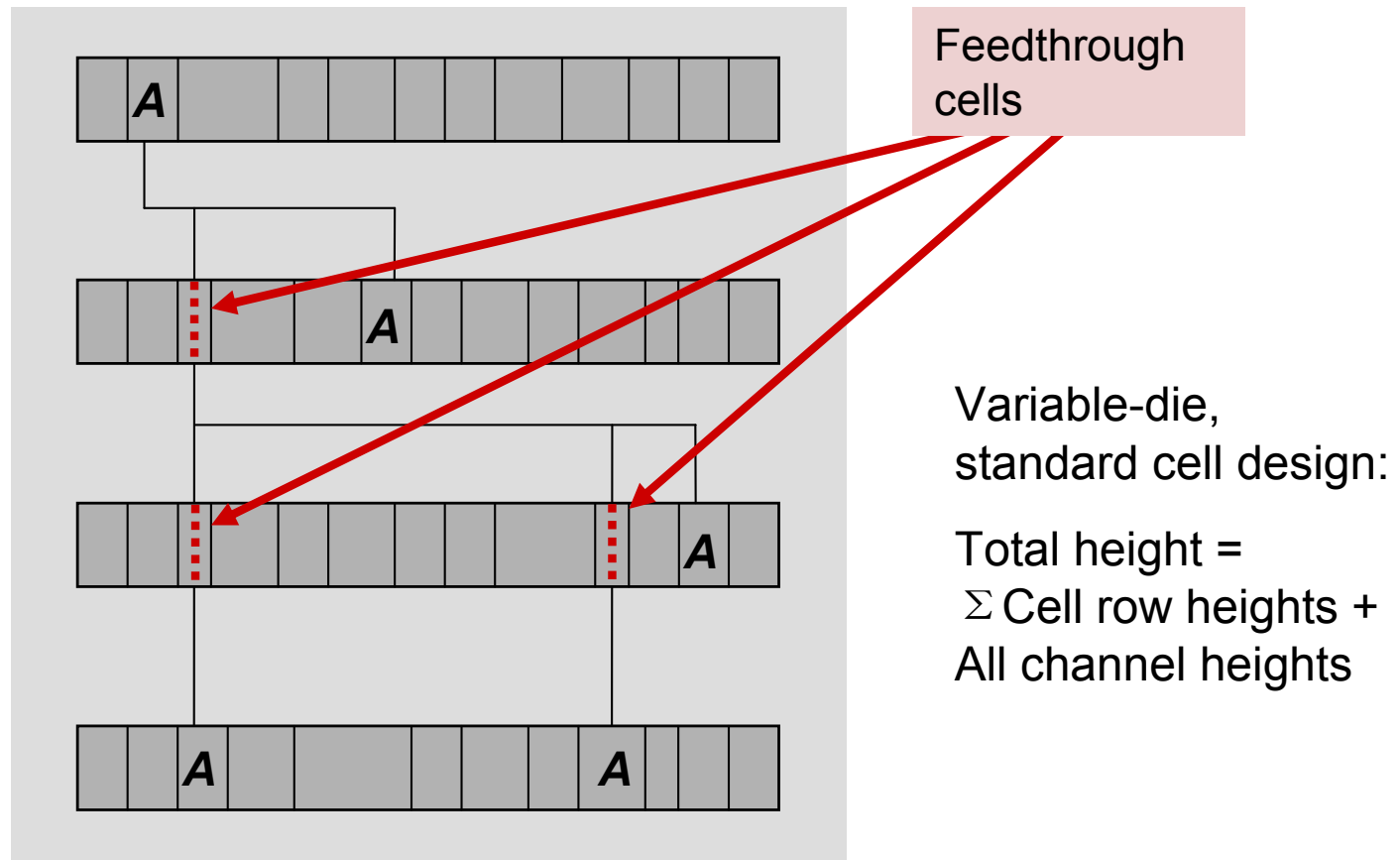


Adapted from [Kahng'11]

Terminology: Feed-Through Cells

Standard-cell design

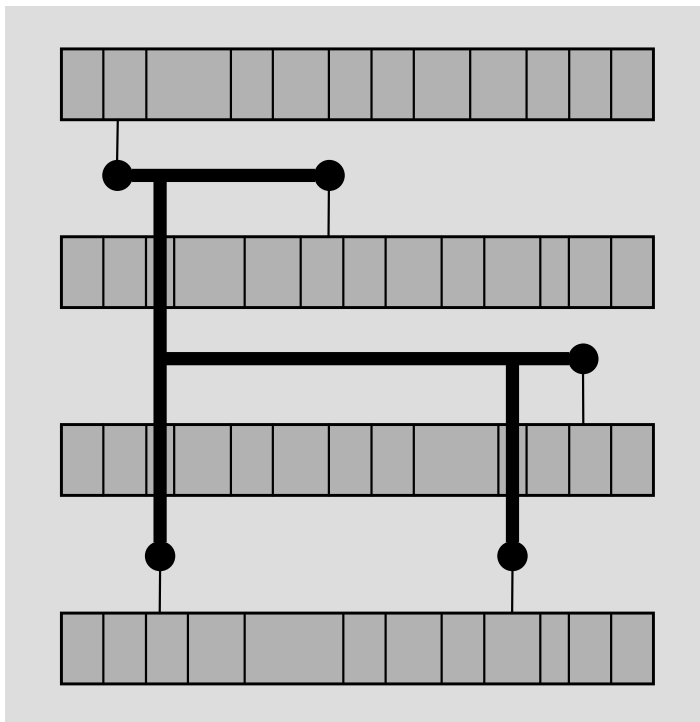
If number of metal layers is limited, feedthrough cells must be used to route across multiple cell rows



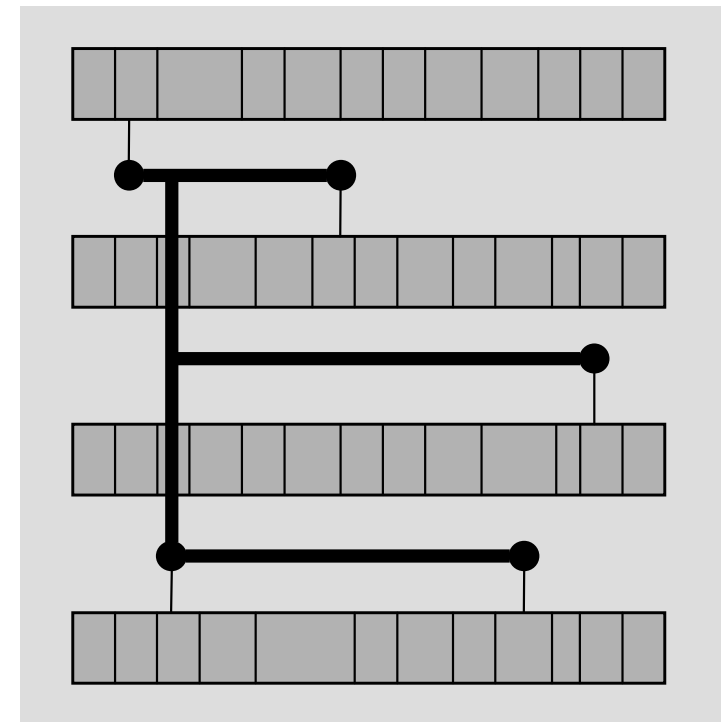
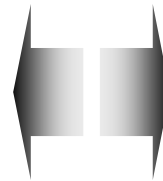
Adapted from [Kahng'11]

Optimizing Wirelength vs. Number of Feedthroughs

Standard-cell design



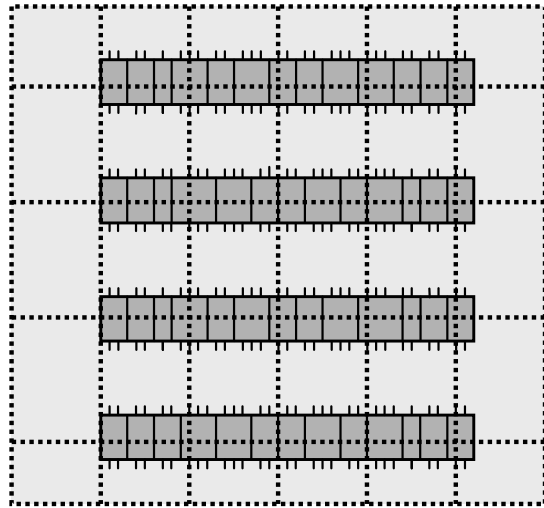
Steiner tree solution with minimal wirelength



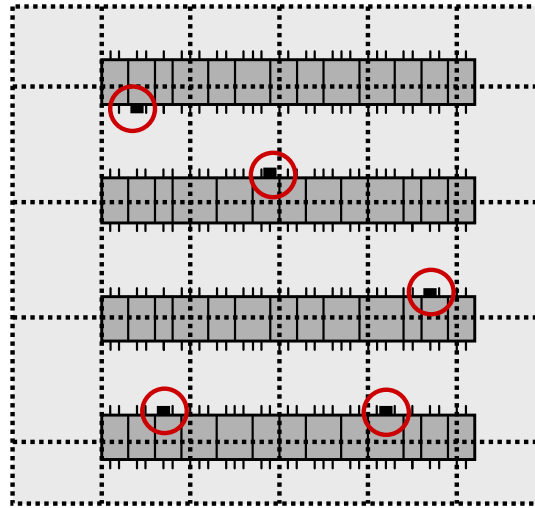
Steiner tree solution with fewest feedthrough cells

Adapted from [Kahng'11]

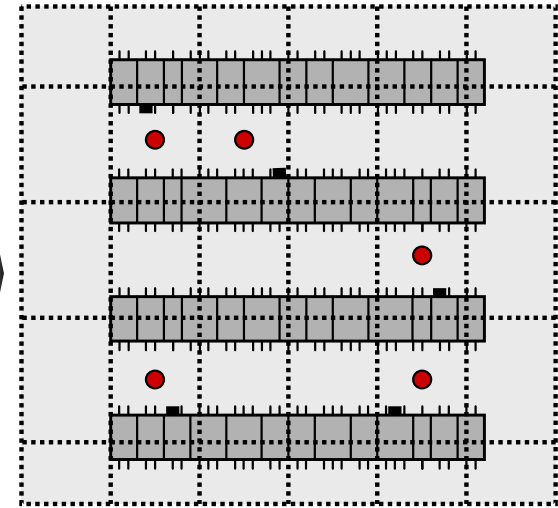
Rectilinear Routing



Defining routing regions



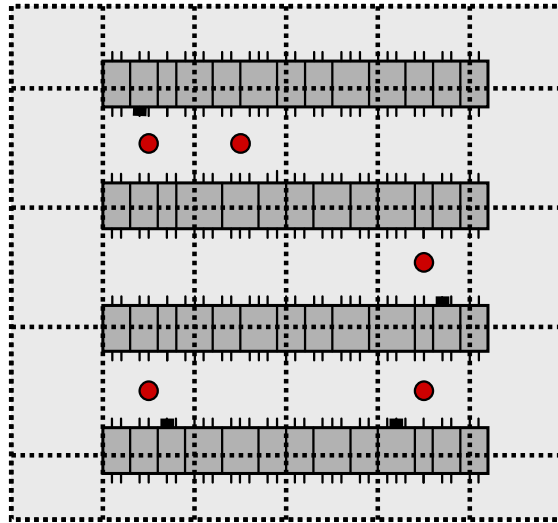
Pin connections



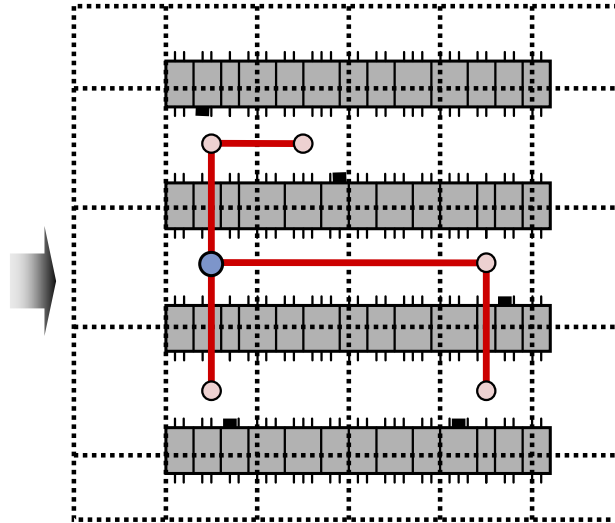
Pins assigned to grid cells

Adapted from [Kahng'11]

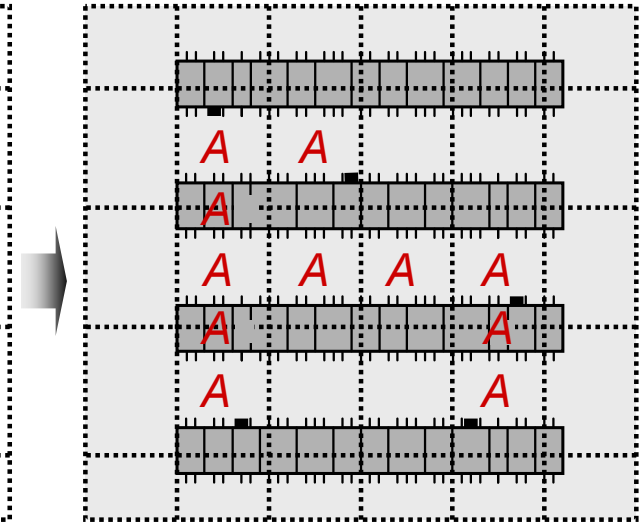
Rectilinear Routing



Pins assigned to grid cells



Rectilinear Steiner minimum tree (RSMT)



Assigned routing regions and feedthrough cells

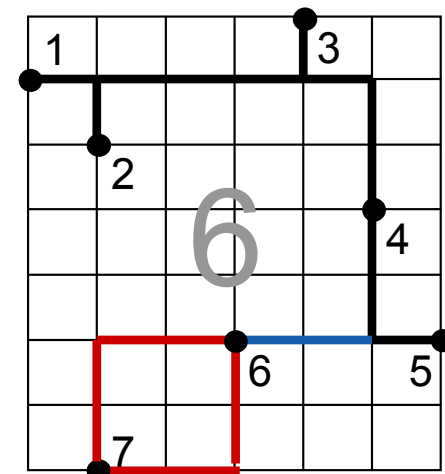
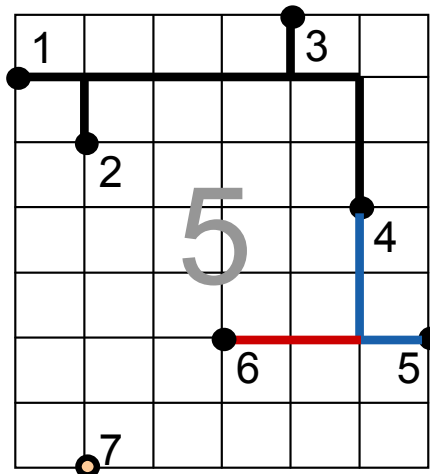
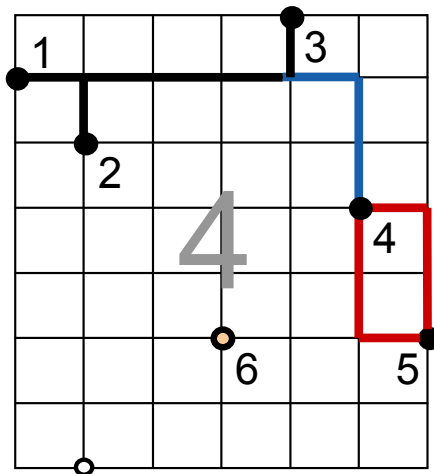
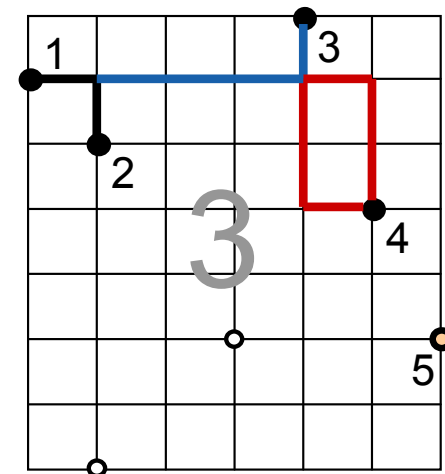
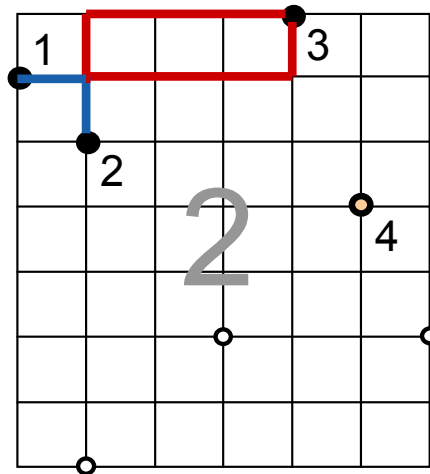
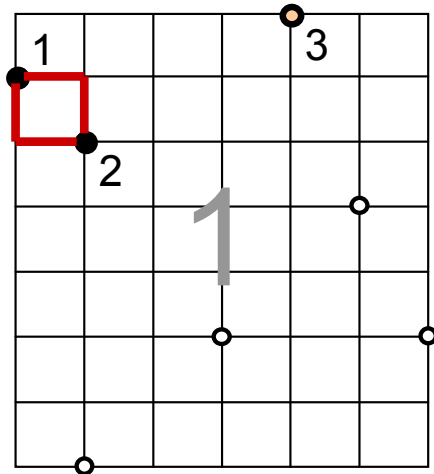
Adapted from [Kahng'11]

Heuristic Sequential Steiner Tree Algorithm

1. Find the closest (in terms of rectilinear distance) pin pair, construct their minimum bounding box (MBB)
2. Find the closest point pair (p_{MBB}, p_C) between any point p_{MBB} on the MBB and p_C from the set of pins to consider
3. Construct the MBB of p_{MBB} and p_C
4. Add the L -shape that p_{MBB} lies on to T (deleting the other L -shape). If p_{MBB} is a pin, then add any L -shape of the MBB to T .
5. Goto step 2 until the set of pins to consider is empty

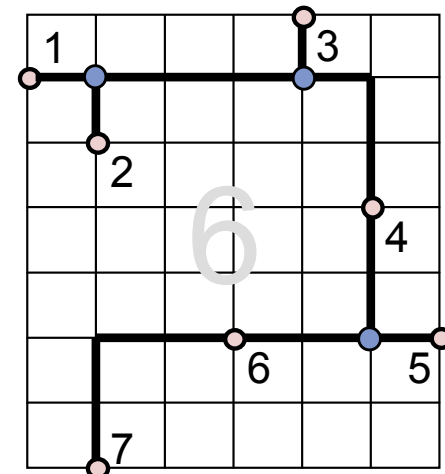
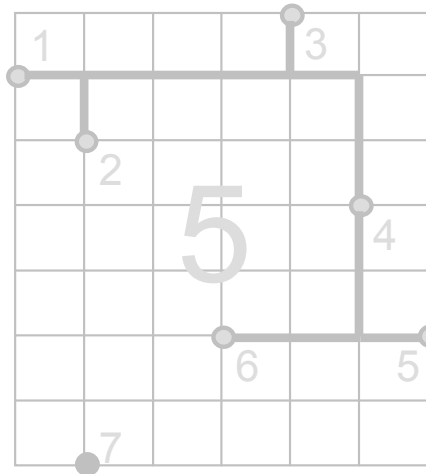
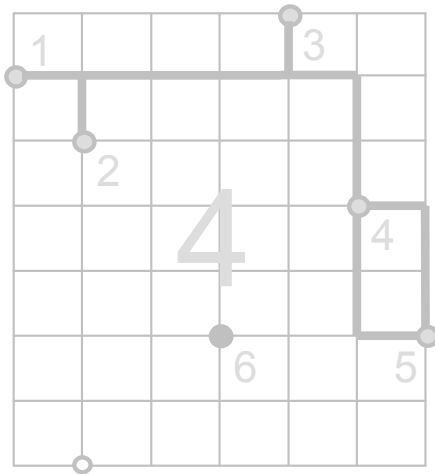
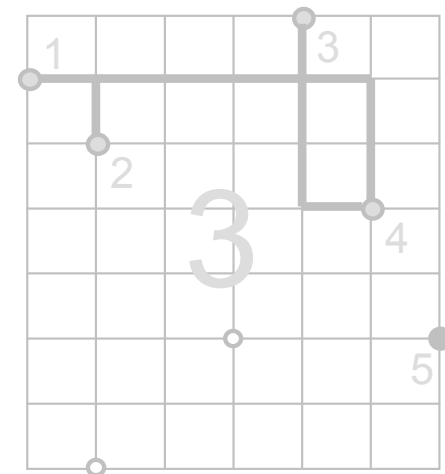
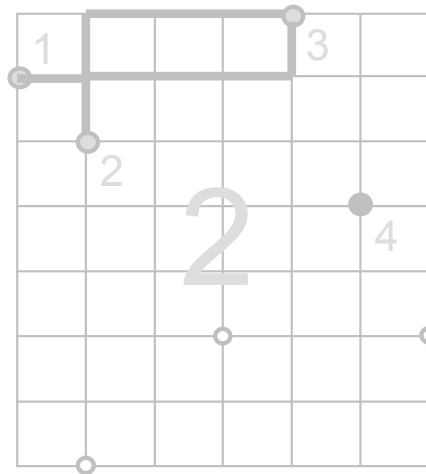
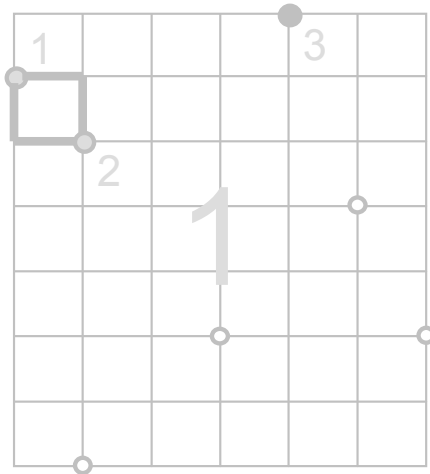
Adapted from [Kahng'11]

Heuristic Sequential Steiner Tree Example



Adapted from [Kahng'11]

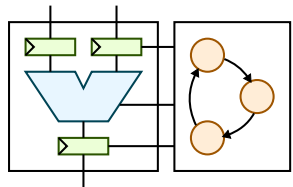
Heuristic Sequential Steiner Tree Example



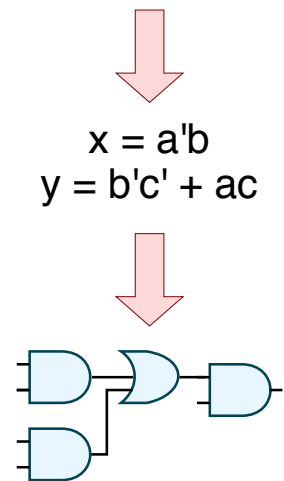
Adapted from [Kahng'11]

Part 3: CAD Algorithms

Topic 12 Synthesis Algorithms



$$x = a'bc + a'bc'$$
$$y = b'c' + ab' + ac$$



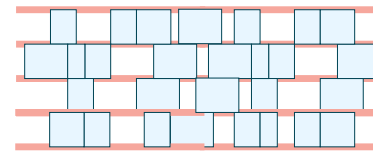
*RTL to Logic
Synthesis*

*Technology
Independent
Synthesis*

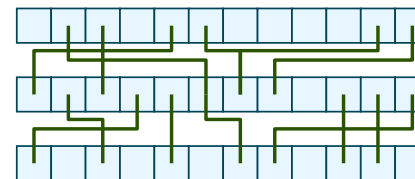
*Technology
Dependent
Synthesis*

Topic 13 Physical Design Automation

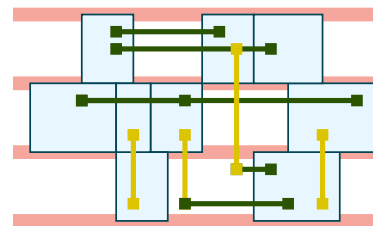
Placement



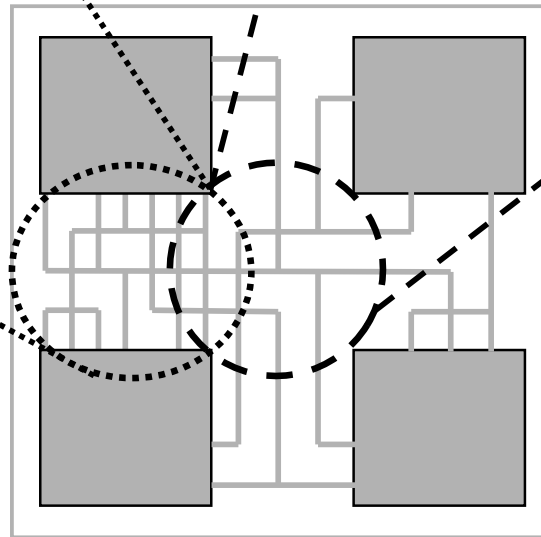
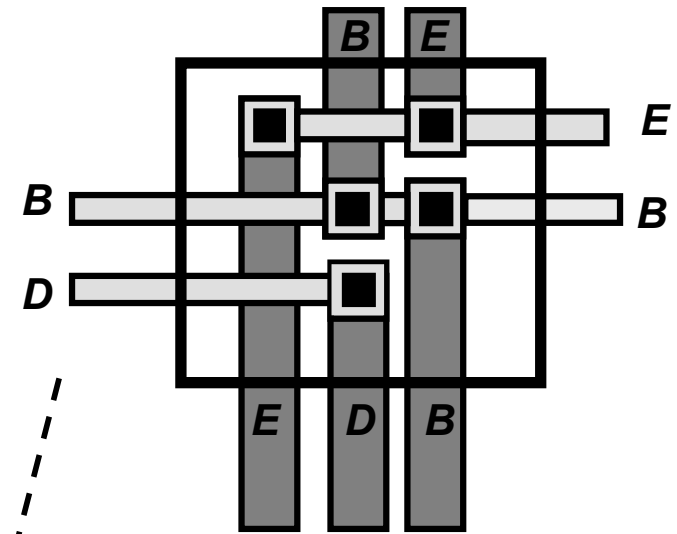
*Global
Routing*



**Detailed
Routing**

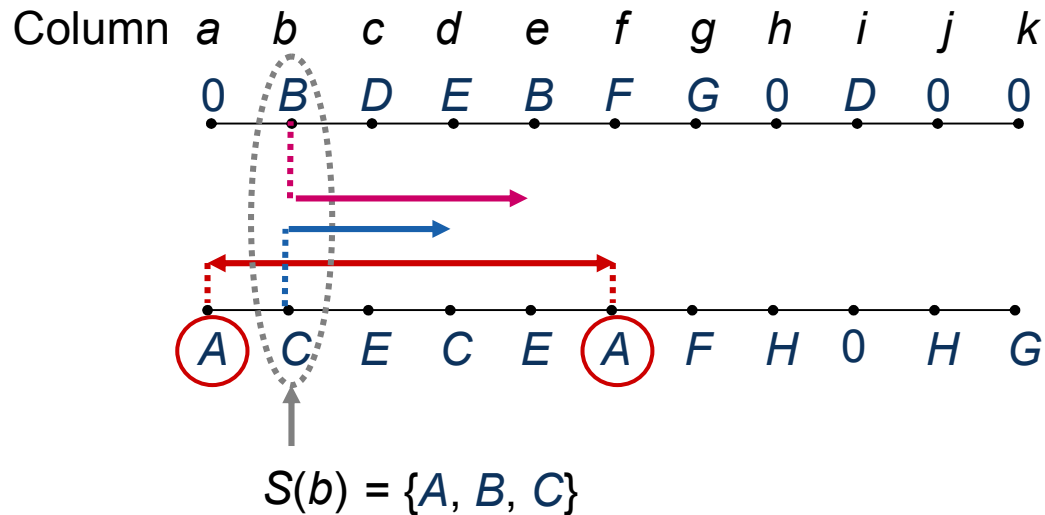


A diagram of a 2D lattice structure. The lattice consists of vertical and horizontal bonds. The vertical bonds are labeled A, B, C, and D. The horizontal bonds are labeled B, C, and D. A red circle highlights a specific region of the lattice, which includes the vertical bonds labeled A, B, C, and D, and the horizontal bonds labeled B, C, and D. The lattice is composed of gray rectangular blocks representing bonds, with some blocks containing a small black square. The labels A, B, C, and D are placed at the ends of the bonds, and the labels B, C, and D are placed above the horizontal bonds. The red circle is drawn around the central part of the lattice, with dotted lines extending from its right side.



Adapted from [Khang'11]

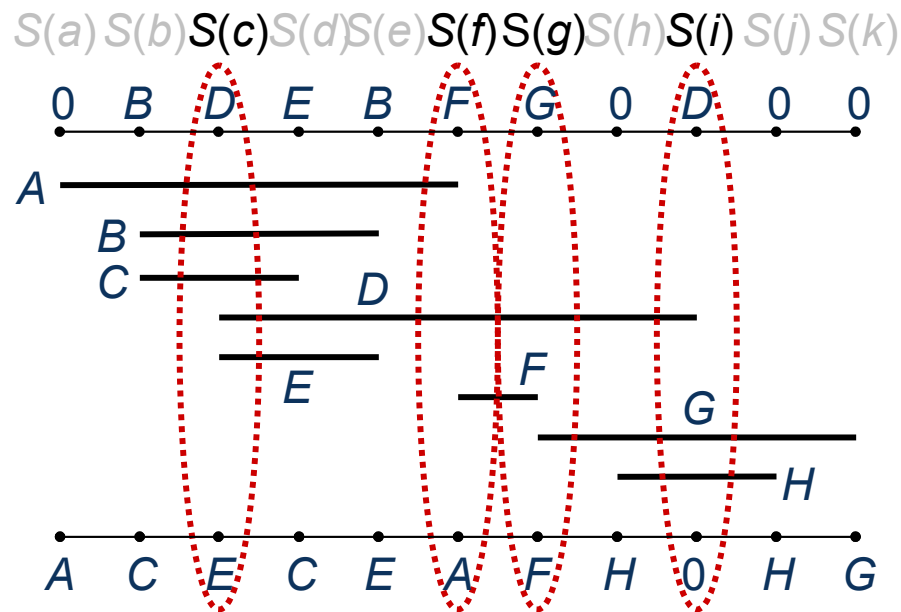
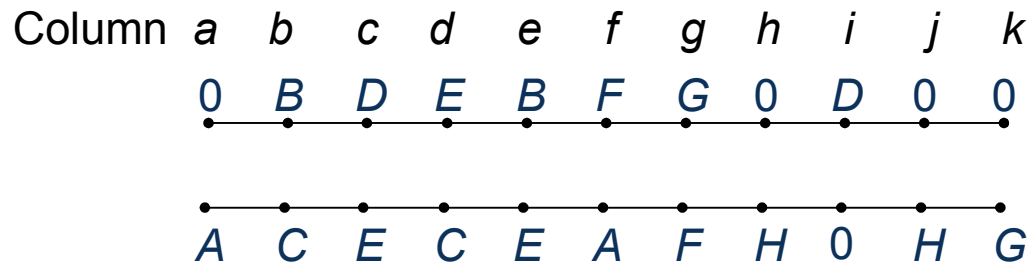
Horizontal Constraint Graphs



- Let $S(col)$ denote the set of nets that pass through column col
- $S(col)$ contains all nets that either (1) are connected to a pin in column col or (2) have pin connections to both the left and right of col
- Since horizontal segments cannot overlap, each net in $S(col)$ must be assigned to a different track in column col
- $S(col)$ represents the lower bound on the number of tracks in column col ; lower bound of the channel height is given by maximum cardinality of any $S(col)$

Adapted from [Khang'11]

Horizontal Constraint Graph Example

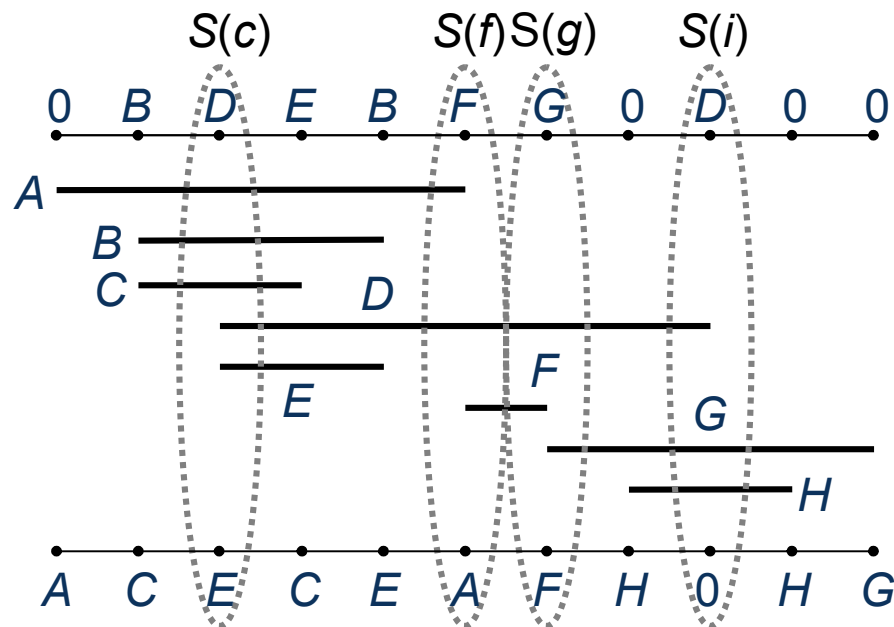
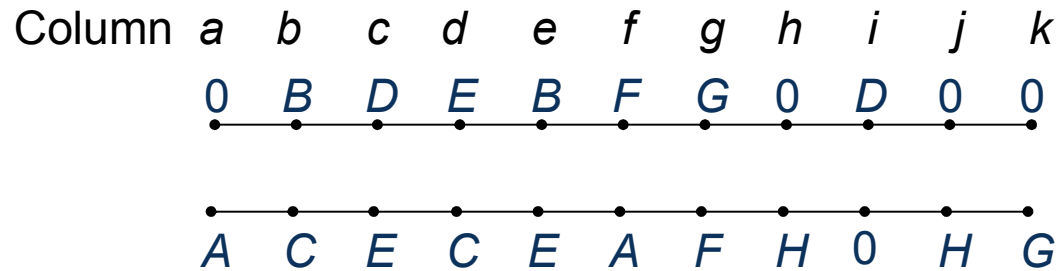


Focus on columns
which are not subsets
of any other column

$S(a) = \{A\}$
 $S(b) = \{A, B, C\}$
 $S(c) = \{A, B, C, D, E\}$
 $S(d) = \{A, B, C, D, E\}$
 $S(e) = \{A, B, D, E\}$
 $S(f) = \{A, D, F\}$
 $S(g) = \{D, F, G\}$
 $S(h) = \{D, G, H\}$
 $S(i) = \{D, G, H\}$
 $S(j) = \{G, H\}$
 $S(k) = \{G\}$

Adapted from [Khang'11]

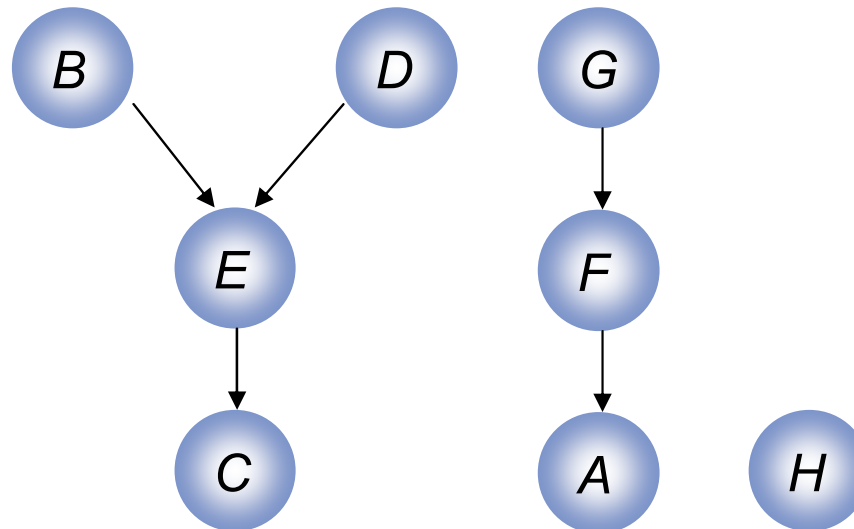
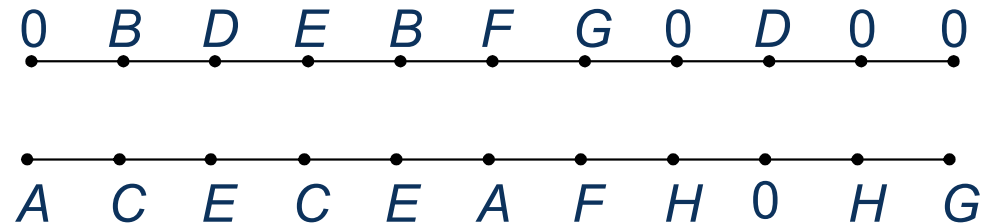
Horizontal Constraint Graph Example



<i>S(c)</i>	<i>S(f)</i>	<i>S(g)</i>	<i>S(i)</i>
<i>A</i>		<i>G</i>	
<i>B</i>	<i>F</i>		<i>H</i>
<i>C</i>			
<i>D</i>			
<i>E</i>			

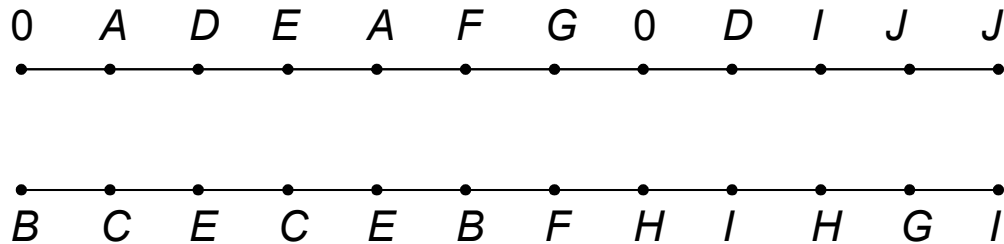
Adapted from [Khang'11]

Vertical Constraint Graphs

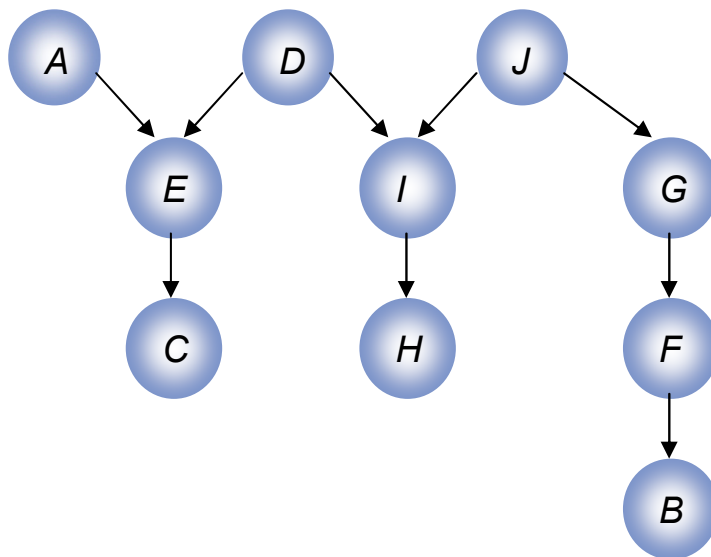


Adapted from [Khang'11]

Left-Edge Algorithm Example



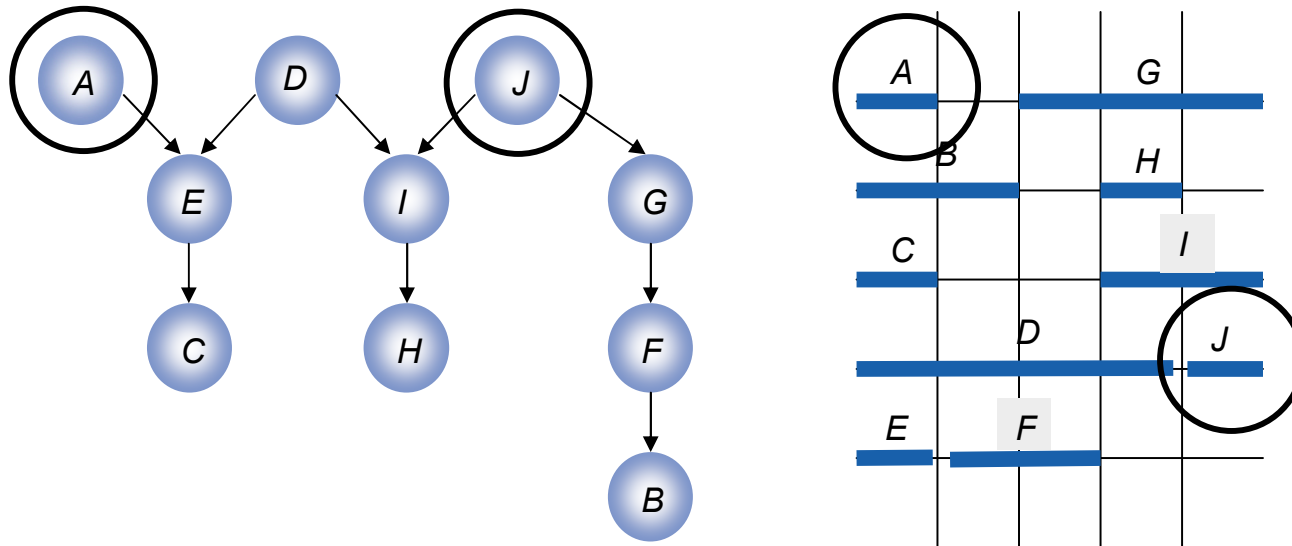
1. Generate VCG and zone representation



A			G	
	B		H	
C				I
		D		J
E	F			

Adapted from [Khang'11]

Left-Edge Algorithm Example



2. Consider next track
3. Find left-to-right ordering of all unassigned nets
If *curr_net* has no parents and does not cause conflicts on *curr_track*
assign *curr_net*

curr_track = 1: Net A Net J

4. Delete placed nets (A, J) in VCG and zone representation

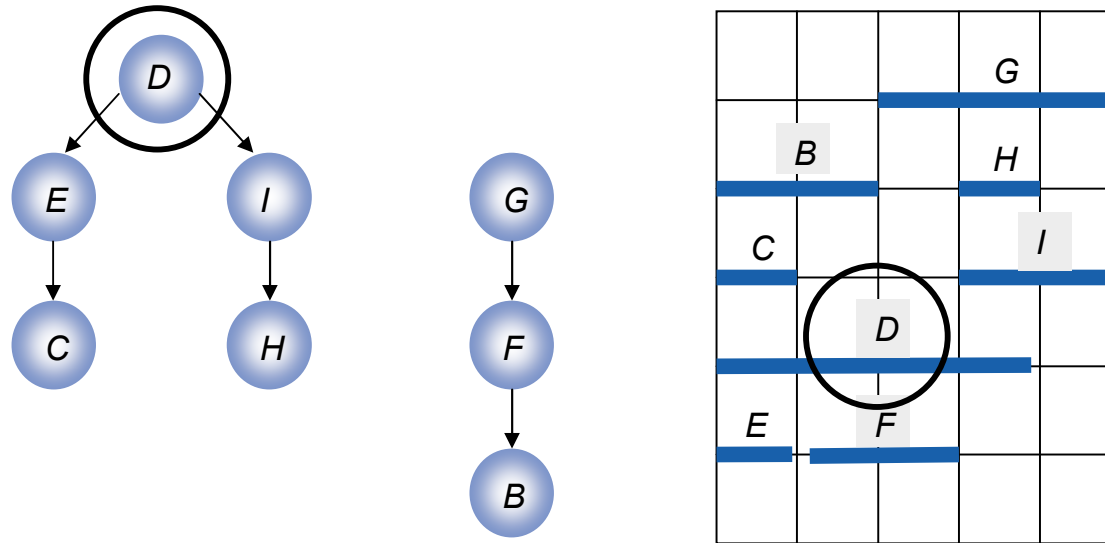
Adapted from [Khang'11]

Left-Edge Algorithm Example



Adapted from [Khang'11]

Left-Edge Algorithm Example



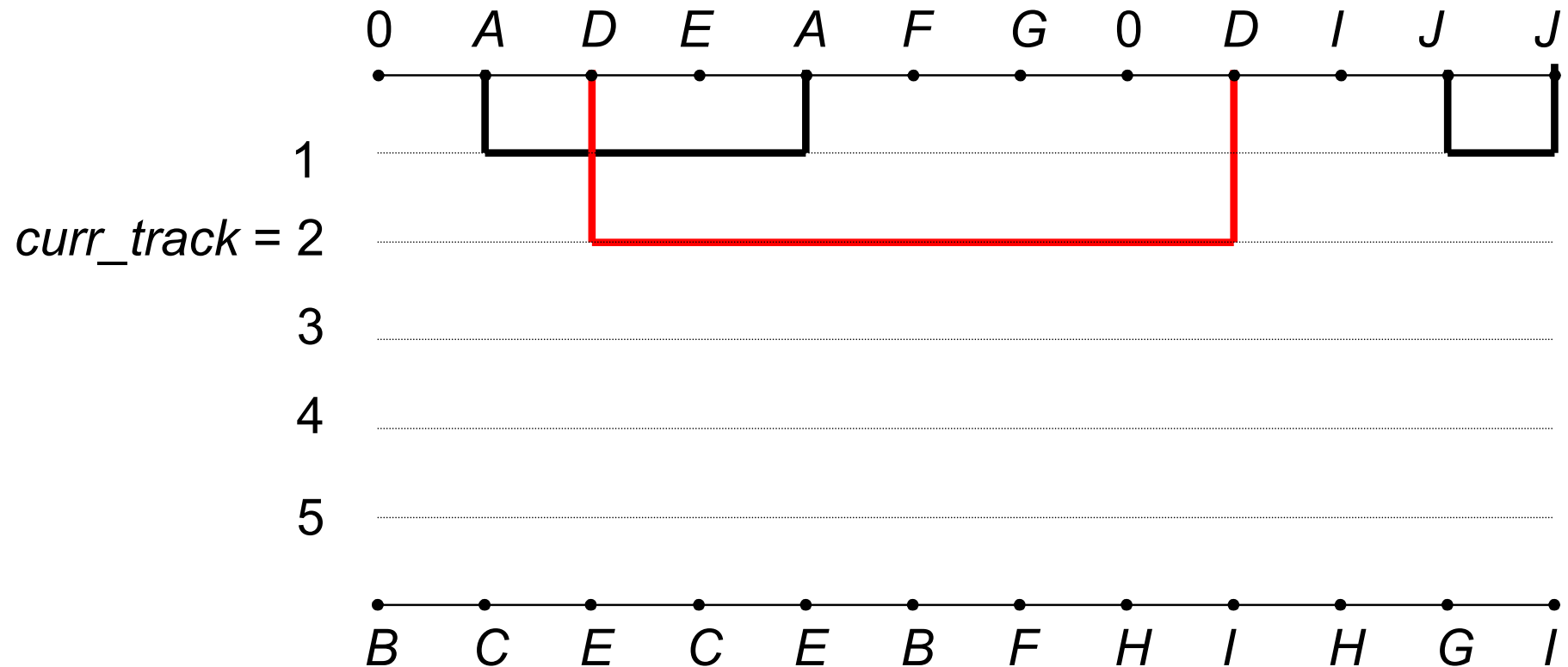
2. Consider next track
3. Find left-to-right ordering of all unassigned nets
If *curr_net* has no parents and does not cause conflicts on *curr_track*
assign *curr_net*

curr_track = 2: Net D

4. Delete placed nets (*D*) in VCG and zone representation

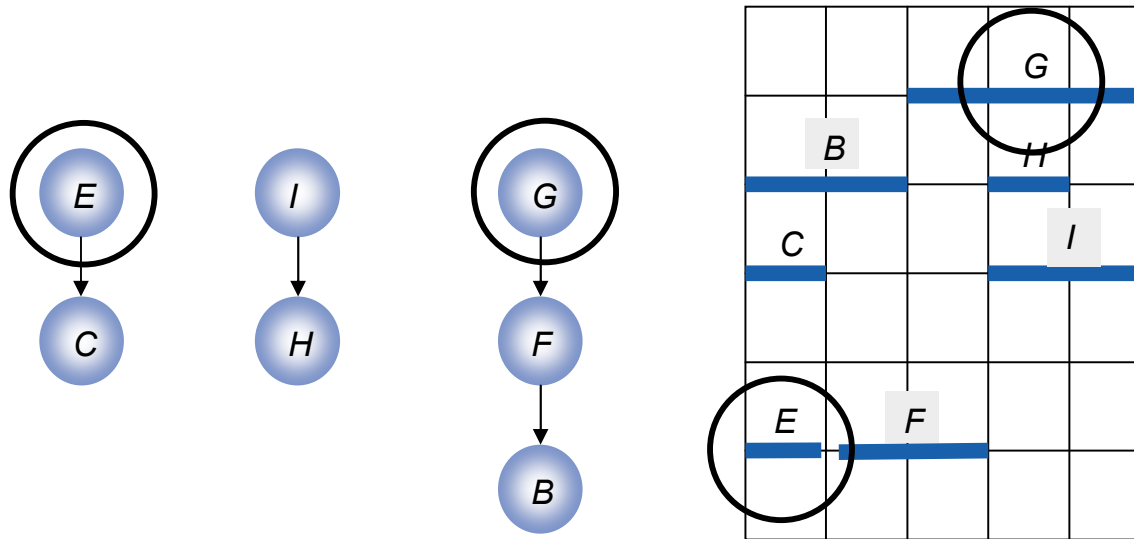
Adapted from [Khang'11]

Left-Edge Algorithm Example



Adapted from [Khang'11]

Left-Edge Algorithm Example



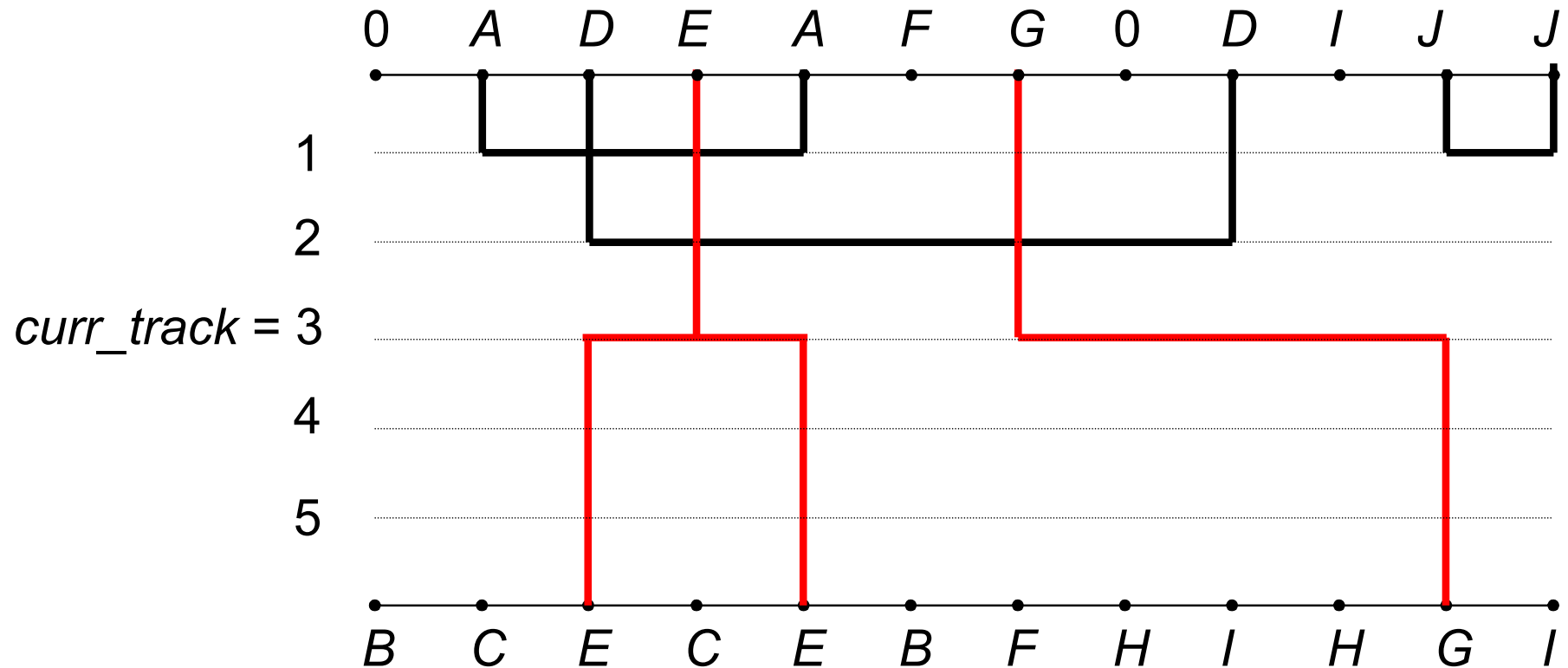
2. Consider next track
3. Find left-to-right ordering of all unassigned nets
If *curr_net* has no parents and does not cause conflicts on *curr_track*
assign *curr_net*

curr_track = 3: Net E Net G

4. Delete placed nets (*E*, *G*) in VCG and zone representation

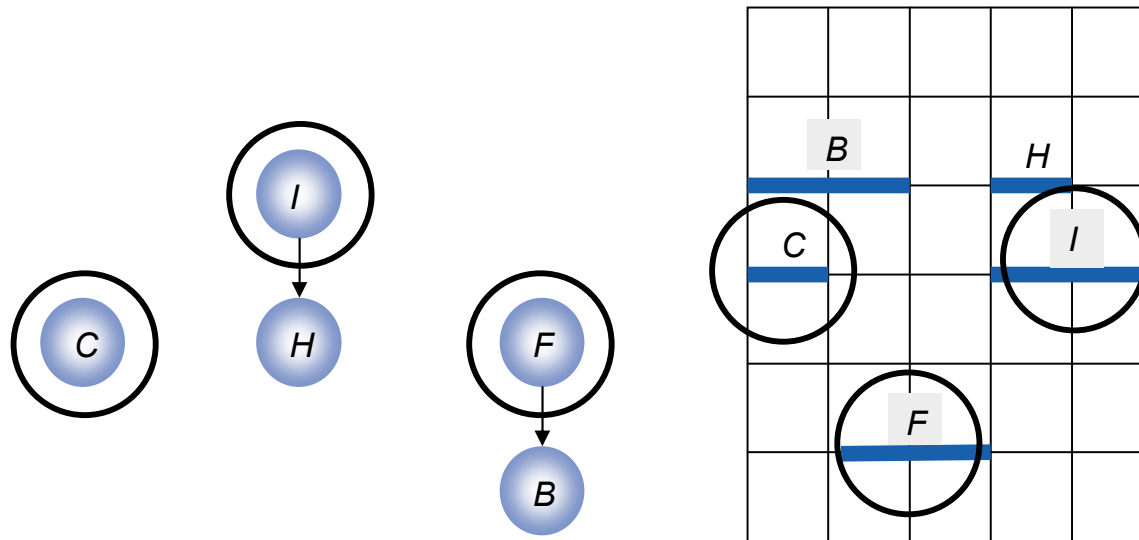
Adapted from [Khang'11]

Left-Edge Algorithm Example



Adapted from [Khang'11]

Left-Edge Algorithm Example



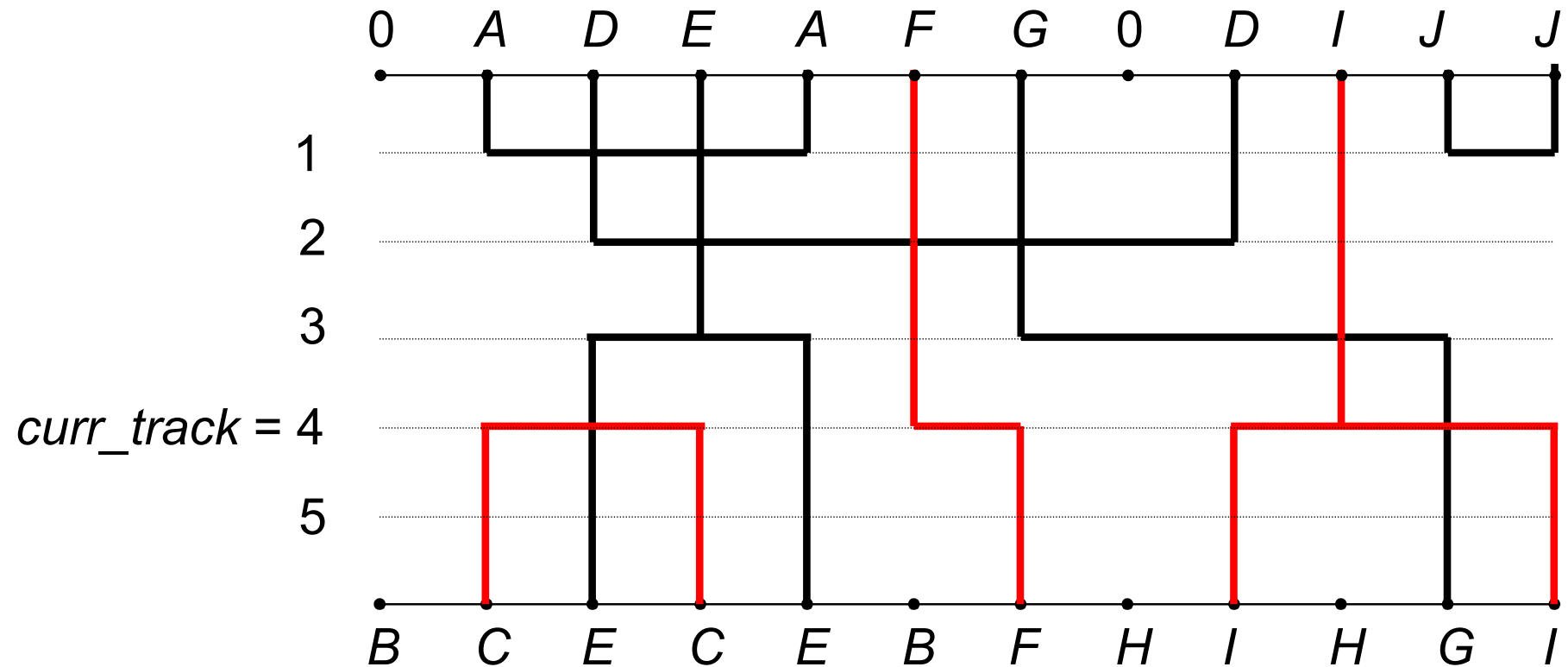
2. Consider next track
3. Find left-to-right ordering of all unassigned nets
If *curr_net* has no parents and does not cause conflicts on *curr_track*
assign *curr_net*

curr_track = 4: Net C Net F Net I

4. Delete placed nets (C, F, I) in VCG and zone representation

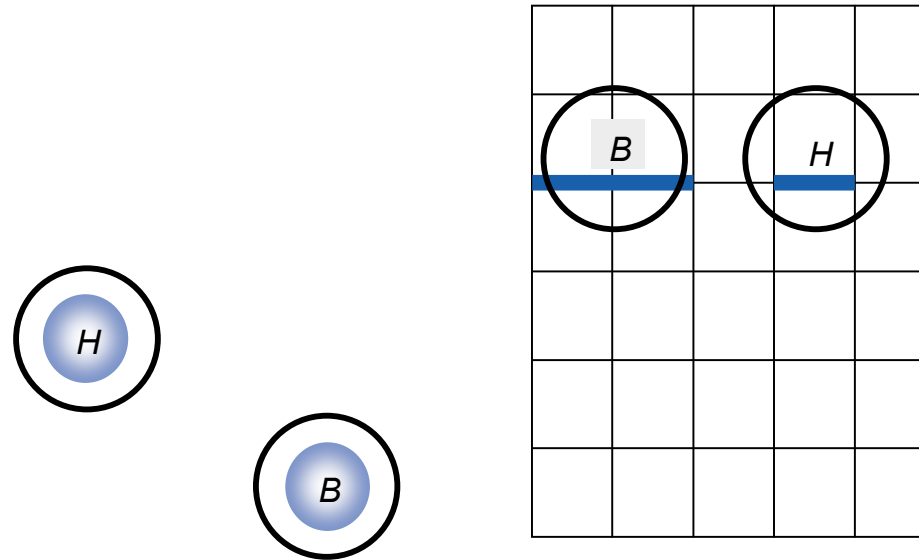
Adapted from [Khang'11]

Left-Edge Algorithm Example



Adapted from [Khang'11]

Left-Edge Algorithm Example



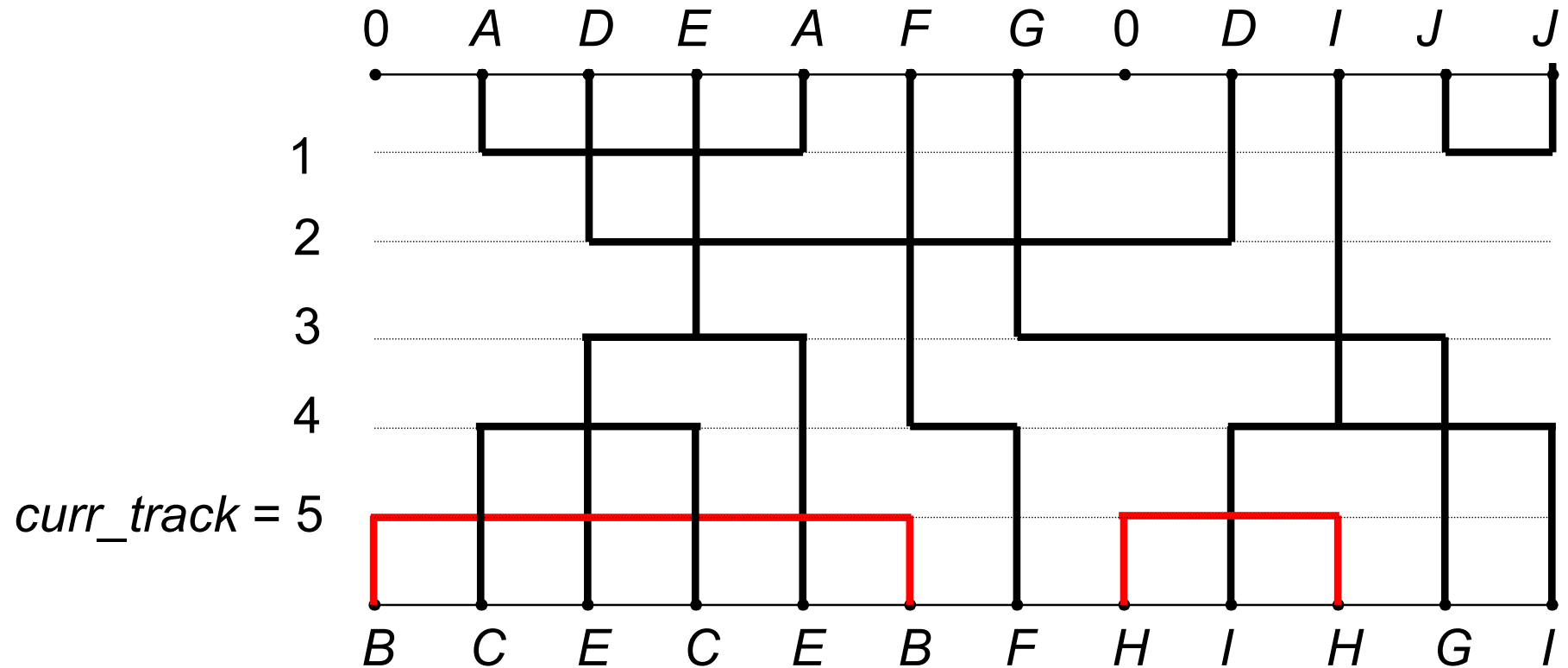
2. Consider next track
3. Find left-to-right ordering of all unassigned nets
If *curr_net* has no parents and does not cause conflicts on *curr_track*
assign *curr_net*

curr_track = 5: Net *B* Net *H*

4. Delete placed nets (*B*, *H*) in VCG and zone representation

Adapted from [Khang'11]

Left-Edge Algorithm Example



Adapted from [Khang'11]

Acknowledgments

- ▶ [Devadas'06] S. Devadas, "VLSI CAD Flow: Logic Synthesis, Placement, and Routing," MIT 6.375 Complex Digital Systems Guest Lecture Slides, 2006.
- ▶ [Kahng'11] A.B. Kahng, J. Liening, I.L. Markov, and J. Hu. Companion Slides for "VLSI Physical Design: From Graph Partitioning to Timing Closure," Springer 2011.