ECE 2400 Computer Systems Programming, Spring 2025
PA1: Math Functions

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Clarifications or fixes to this writeup after the initial publication will appear in this color. (See pp 6 & 7.)

1. Introduction

This programming assignment is a warmup designed to give you experience with two important aspects of computer systems programming: software design and software testing. You will leverage the basic concepts from lecture, ranging from variables and operators to conditional and iteration statements. More advanced concepts such as recursion will also play a key role in optimizing the performance of your code.

You will implement the *square root* math function twice: first using a simple but naive implementation, and then using a more sophisticated algorithm that can significantly improve performance.

After your code is functional and tested, you will write a four-page report that includes a discussion of your testing strategy and a quantitative evaluation of the performance across three implementations. **You should consult the programming assignment logistics document** for more information about the expectations for all programming assignments and how they will be assessed. While the final code and report are all due at the end of the assignment, **we also require meeting an incremental milestone in this PA.** Requirements specific to this PA for the incremental milestone and the final submission are described at the end of this handout.

This handout assumes that you have read and understand the course tutorials and that you have attended the discussion sections. To get started, log in to an `ecelinux` server, and clone your individual remote repository from GitHub:

```
% mkdir -p ${HOME}/ece2400
% cd ${HOME}/ece2400
% git clone git@github.com:cornell-ece2400/netid
% cd ${HOME}/ece2400/netid/pa1-math
```

...where `netid` should be replaced with your NetID. You can both pull and push to your individual remote repository. **You should never fork your individual remote repository! If you need to work in isolation then use a branch within your individual remote repository.**

First things first, the `README` file in your `pa1-math` directory tells you how to get started. You can view the `README` on your computer, or you can find it online at

`https://github.com/cornell-ece2400/netID/tree/main/pa1-math#readme`

... this will give you a 404 Error until you replace `netid` in the URL with your NetID.

Aside: The web view of your repository is a great way to see what WE see, when we look at your code. If your code online looks different than the code on your computer, that means you haven't committed your changes.

The `pa1-math` subproject has many files and sub-folders. If you run the command `tree` from within the `pa1-math` directory, you'll see the entire directory structure:

```
[netID@ecelinux-16 pa1-math]$ tree
.
├── build.sh                          script to compile your code
├── CMakeLists.txt                    one of many configuration files
├── eval                              timing evaluation files go here
│   ├── CMakeLists.txt
│   ├── sqrt-iter-eval.c              programs that evaluate the iterative,
│   ├── sqrt-recur-eval.c             recursive, and standard C library
│   └── sqrt-std-eval.c               implmentation of sqrt
├── eval.sh                           builds, tests, and then evaluates
├── include                           header files go here
│   ├── ece2400-stdlib.h              whichever header file a program
│   ├── sqrt-iter.h                   imports determines which
│   └── sqrt-recur.h                  implementation gets used
├── README.md                         How to start the project
├── src                               source code goes here
│   ├── CMakeLists.txt
│   ├── ece2400-stdlib.c              Your 2 implementations
│   ├── sqrt-iter.c                   of sqrt live here
│   ├── sqrt-iter-main.c
│   ├── sqrt-recur.c                  These programs are the simplest
│   └── sqrt-recur-main.c             way to run your implementations.
├── test                              test files go here
│   ├── CMakeLists.txt
│   ├── sqrt-iter-directed-test.c
│   ├── sqrt-iter-random-test.c
│   ├── sqrt-recur-directed-test.c
│   └── sqrt-recur-random-test.c
└── test.sh                           builds and tests your code
4 directories, 22 files
```

Throughout the entire directory structure are multiple configuration files called `CMakeLists.txt`. These are used to generate the Makefile that is used to compile the source code. You won't need to modify these, but if you take a peek at them (`less CMakeLists.txt`, then `q` to quit `less`) you can see how they relate to the contents of each directory.

2. Interface and Implementation Specifications

You will be implementing two different algorithms to compute the *square root* (`sqrt`) math function.

`sqrt` Interface

Regardless of implementation, the `sqrt` function will have the following prototype:

```
int sqrt( int x );
```

The function takes one input argument (x) and returns its square root (i.e., $\sqrt{x}$). Notice that the variant of `sqrt` that we will use in this assignment takes an integer input and returns another integer. The return value is the square root of x rounded down to the nearest integer. For example, calling `sqrt(5)` will return `2`. Your implementations should work correctly when the input is zero and when the input is any valid positive integer. If x is a negative value, the `sqrt` function must return -1 to report an invalid input.

Your implementations cannot use anything from the Standard C library except for the `printf` function defined in `stdio.h`, the MIN/MAX macros defined in `limits.h`, and the `assert` macro defined in `assert.h`. Your implementations should not use any floating point arithmetic since this can introduce unnecessary performance overhead. Your implementations should not use any magic numbers. Note that you *can* use `INT_MAX` and `INT_MIN` from `limits.h` if you need to determine the largest and smallest value that can be stored in a variable of type `int`.

Iterative `sqrt` Implementation

The iterative implementation of the `sqrt` function should be implemented using an iteration statement. Let i range from zero to x. For each i, compute $i \times i$ and compare the result with x. If $i \times i$ is smaller than x, then i is less than the square root of x. If $i \times i$ is larger than x, then i is greater than the square root of x. By gradually checking all values of i, you will be able to find the square root of x rounded down to the nearest integer. Write your iterative implementation for `sqrt` inside of `src/sqrt-iter.c`.

Implement and fully test your iterative algorithm before moving on to the recursive one.
This is Milestone 1!

Recursive `sqrt` Implementation

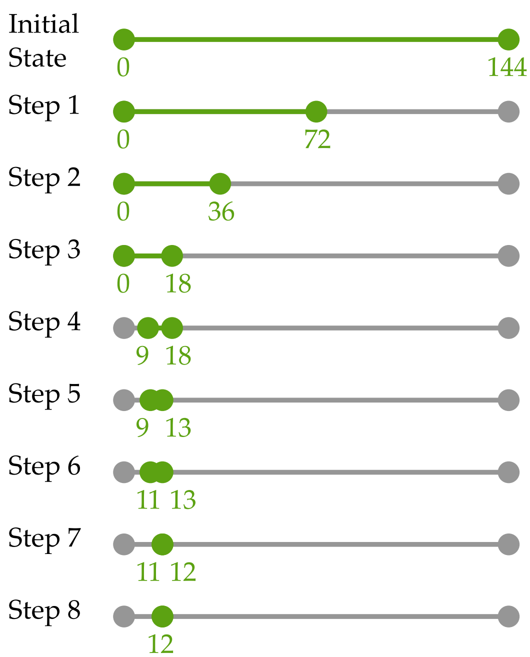
The iterative implementation of `sqrt` is particularly slow when x is large because: (1) the computer executes multiplication operations more slowly compared to simpler operations (e.g., addition, subtraction); and (2) the number of multiply operations increases linearly with $\sqrt{x}$ since we are doing an exhaustive search. We can use a more sophisticated search to reduce the number of multiply operations. Consider the situation when x is 144. We can divide the search space into two ranges:

- Range of integers from 0 to $\frac{x}{2}$, which is $[0, 72]$ when x is 144
- Range of integers from $\frac{x}{2}$ to x, which is $[72, 144]$ when x is 144

We can quickly determine which half the square root of x lies in by squaring the midpoint (i.e., $72 \times 72 = 5184$) and comparing to x. Observing $5184 > 144$ tells us that our guess of 72 was much too high, so the answer must be in the lower half (i.e., somewhere in the range $[0, 72]$), which is true since we know in this example that the square root is 12. We can continue applying the same approach on the smaller range, dividing the search space into smaller and smaller ranges. Figure 1 illustrates an example execution when x is 144. This approach allows us to quickly "zoom in" on the square root of x. We can capture this algorithm iteratively, but a recursive solution is also possible and may be more elegant and concise. The general approach of repeatedly halving the search space is known as a *binary search*. We will learn more about this class of algorithms in the future. Write your recursive implementation for `sqrt` inside of `src/sqrt-recur.c`. You may add additional helper functions inside this file as needed.

$$b^e = \begin{cases} 1 & \text{if } e = 0 \\ b \times b \times \overset{e}{\cdots} \times b & \text{if } e > 0 \end{cases}$$

$$b^e = \begin{cases} 1 & \text{if } e = 0 \\ b & \text{if } e = 1 \\ (b^2)^{e/2} & \text{if } e > 1 \text{ and } e \text{ is even} \\ b \times b^{e-1} & \text{if } e > 1 \text{ and } e \text{ is odd} \end{cases}$$



- Split the range [0, 144] into [0, 72] and [72, 144]
- Choose the lower range
- Split the range [0, 72] into [0, 36] and [36, 72]
- Choose the lower range
- Split the range [0, 36] into [0, 18] and [18, 36]
- Choose the lower range
- Split the range [0, 18] into [0, 9] and [9, 18]
- Choose the upper range
- Split the range [9, 18] into [9, 13] and [13, 18]
- Choose the lower range
- Split the range [9, 13] into [9, 11] and [11, 13]
- Choose the upper range
- Split the range [11, 13] into [11, 12] and [12, 13]
- Choose the lower range
- Search the range [11, 12]
- Identify the square root of 144 to be 12

Figure 1: Example of Recursive `sqrt` Algorithm – The range of integers that can contain the square root is halved at each step.

3. Testing Strategy

You must develop an effective testing strategy to ensure all implementations are correct. Writing tests is one of the most important and challenging aspects of software programming. Software engineers often spend more time implementing tests than they do implementing the actual program.

Although there are limitations on what you can use from the Standard C library in your *implementations*, there are no limitations on what you can use from the Standard C library in your *testing*. Feel free to use the Standard C library in your golden reference models and/or for random testing.

3.1. Smoke Testing

The `test.sh` script builds *all* the executables in your project and then runs the entire test suite on both of your implementations `sqrt`. When you are first developing your code, this is a bit overkill.

To help students test as they code, we provide a smoke test program for each implementation: `sqrt-iter-main.c` and `sqrt-recur-main.c`. These are simple programs that call the `sqrt-root` function defined in `src/sqrt-iter.c` and `src/sqrt-recur.c`, respectively.

Read the comments at the top of `src/sqrt-iter-main.c` to see how to compile and run it.

The *second* you think you have a working version of iterative square root, run `sqrt-iter-main`.

Same goes for `sqrt-recur-main.c`.

3.2. Systematic Unit Testing

Although a simple smoke test helps you quickly see results of your implementations, it will not provide thorough testing. We need a systematic and automatic unit testing strategy to (hopefully) test all possible scenarios efficiently.

For each implementation, we provide a directed test program that should include several test cases to target different categories and a random test program that should test that your implementation works for random inputs. **We provide only a very few directed tests and no random tests. You must add many more directed and random tests to thoroughly test your implementations!**

A **directed** test case involves manually pre-computing the output for a specific set of inputs, and then verifying that your implementation produces this desired output. Start by writing as many directed test cases as you can for some simple and more complex inputs. As you design your implementations, pay careful attention to edge cases and unexpected inputs (e.g., negative inputs) that break the functionality of your code. When you encounter such a case, capture the situation with a directed test case and verify your implementation now passes that test case. Carefully read the implementation specification (i.e., the inputs, the outputs, and the behavior), so you know how your program should respond in all possible scenarios. Convince yourself that your implementations are *robust* by carefully developing a testing *strategy*.

You should also add **random** tests to increase your confidence in the correctness of your implementation. You can randomly generate inputs using the `rand` function in the standard C library (include `stdlib.h`). Use the `srand` function to initialize the random seed to a deterministic value to ensure your random tests are repeatable. You can use the `sqrt` function in the standard C library (include `math.h`) as a golden reference model to generate correct reference outputs which you can then compare to the results from your own implementations. Note that you are not allowed to use the `sqrt` function in the standard C library for your implementation, only for testing.

We provide you a testing framework you should use for your directed and random testing. See the provided test programs in the `test` subdirectory for how to use this framework. The ECE 2400 standard library in `ece2400-stdlib.h` contains the following macros you should use to check the correctness of your implementations:

- `ECE2400_CHECK_FAIL()` – check program does not reach this point
- `ECE2400_CHECK_TRUE( expr_ )` – check `expr_` is always true
- `ECE2400_CHECK_FALSE( expr_ )` – check `expr_` is always false
- `ECE2400_CHECK_INT_EQ( expr0_, expr1_ )` – check `expr0_` equals `expr1_`

The `README` file in your `pa1-math` directory shows you how to create a `build` directory and use the `build.sh` script to compile the code. Running this script creates multiple executable files (i.e, programs) that reside in the `build` directory. Once you do this, you can run all unit tests for all implementations via the `test.sh` script, which builds and tests your code:

```
 % ./test.sh
 <blah blah blah>
 Build Successful
 <blah blah blah>
Tests failed
 %
```

(Your tests will all fail initially.)

If you are failing a test program, then you can "zoom in" and run all of the unit tests for a single test program (e.g., directed tests for `sqrt-iter`) like this:

```
% cd ${HOME}/ece2400/netid/pa1-math/build/test
% make sqrt-iter-directed-test
% ./sqrt-iter-directed-test
```

You can then "zoom in" to a specific test case by passing in the index of that test case like this:

```
% cd ${HOME}/ece2400/netid/pa1-math/build/test
% make sqrt-iter-directed-test
% ./sqrt-iter-directed-test 1
test_case_1_simple
- [ FAILED ] sqrt-iter-directed-test.c:20:  sqrt_iter(0) != 0 (-1 != 0)

% ./sqrt-iter-directed-test 2
test_case_2_negative
- [ passed ] sqrt-iter-directed-test.c:31:  sqrt_iter(-10) == -1 (-1 == -1)
%
```

3.3. Test-Case Crowd Sourcing

While a comprehensive test suite provides strong evidence that your implementation has the correct functionality, it is particularly challenging to write high-quality test cases for all of your implementations. After the milestone deadline, **students can use test-case crowd-sourcing to reduce the workload of constructing a comprehensive test suite.** Test-case crowd-sourcing will use a Canvas discussion page; students cannot see any of the currently posted test cases until they post one of their own. Focus on uploading one or two very strong directed or random test cases. Do not upload more than two test cases. Avoid uploading simple directed test cases since students will have already developed such test cases for the milestone. Posting the basic test case provided by the course instructors, posting an obviously too simple test case, and/or posting something which is obviously meant to "game" the system is not allowed. Let's all work together to crowd-source a great test suite that every student can take advantage of!

You can use test cases posted in the Canvas discussion page in your test programs as long as you acknowledge the author, so be sure to include the comment in your source code which describes the test case and includes the author's name. You will need to renumber the test cases and call them correctly from `main()`. **Make sure you understand the test case and that you feel it is testing correct behavior before including it in your test suite!**

4. Evaluation

Once you have tested the functionality of the iterative and recursive implementations, you can then start to evaluate the performance of these implementations. We provide you an evaluation program for each implementation in the `eval` subdirectory. In addition, we also provide you an evaluation program for the `sqrt` function provided in the standard `math` library. You should not need to modify the evaluation programs. The ECE 2400 standard library in `ece2400-stdlib.h` contains the following functions that are used in the evaluation programs to measure the execution time:

- `ece2400_timer_reset()` – reset global timer
- `ece2400_timer_get_elapsed()` – return elapsed time in seconds since last reset

You can run all the evaluation programs by running the `eval.sh` script.

```
 % cd ${HOME}/ece2400/netid/pa1-math
 % ./eval.sh
```

If you run this before you have implemented and tested your work, the script will likely end with the following message:

```
 % ./eval.sh
<blah blah blah>
<error error error>
Tests failed
SKIPPED: evaluation
%
```

This is because it does not make sense to assess the performance of an incorrect program. The speed of an incorrect implementation is useless.

To run an individual evaluation, simply specify the inputs on the command line. For example, the following runs an evaluation for one of the `sqrt` implementations to find the square root of 100.

```
 % cd ${HOME}/ece2400/netid/pa1-math/build/eval
 % make
 % ./sqrt-std-eval 100
Evaluating sqrt of 100
Elapsed time for trial 0 is 0.006902s
Elapsed time for trial 1 is 0.006928s
Elapsed time for trial 2 is 0.006887s
Elapsed time for trial 3 is 0.006878s
Elapsed time for trial 4 is 0.006890s
Elapsed time (averaged) is 0.006897s
Verification passed! sqrt yields 10 and ref is 10
%
```

The evaluation programs apply your math functions to the input you specify at the command line in a loop and report the total wall-clock runtime. This will enable you to compare the performance between your iterative algorithms, recursive algorithms, and the implementations provided in the standard `math` library. The evaluation programs also ensure that your implementations are producing the correct results, however, you should not use the evaluation programs for testing. If your implementations fail during the evaluation, then your testing strategy is insufficient. You must add more unit tests to effectively test your program before returning to performance evaluation.

You should quantitatively evaluate all three evaluations for a range of values. We suggest evaluating your `sqrt` implementations from zero to one million with a reasonable number of intermediate points as long as the implementation doesn't take too long to run. Record all of this performance data.

5. Milestone and Final Submission

This section includes critical information about the incremental milestone, final code submission, and the final report specific to this PA. **The programming assignment logistics document provides**

general details about the requirements for the milestone and final submission. You must actually read the document to ensure you know how we will access your milestone and final submission.

5.1. Incremental Milestone

While the final code and report are all due at the end of the assignment, we also require you to complete an incremental milestone and push your code to GitHub by the date specified by the instructor. In this PA, to meet the incremental milestone, you will need to (1) complete the iterative implementation of `sqrt` and (2) write an extensive test suite including many directed and random tests for this implementation.

5.2. Final Code Submission

Your code quality score will be based on the way you format the text in your source files, proper use of comments, deletion of instructor comments, and uploading the correct files to GitHub (only source files should be uploaded, no generated build files). Be sure to follow the course C Coding Conventions!

Note that students must remove unnecessary comments that are provided by instructors in the code distributed to students. Students must not commit executable binaries or any other unnecessary files.

To submit your code you simply upload it to GitHub. Your code will be assess both in terms of functionality and code quality. Your functionality score will be determined by running your code against a series of tests developed by the instructors to test its correctness.

5.3. Final Report

The final report must be uploaded to Canvas. The date you upload your report will indicate how many slip days you are using for the assignment. For this PA, we require you to include four sections: introduction, testing strategy, quantitative evaluation, and conclusion. Your entire report must be no more than four pages.

For PA1 we have provided you with a detailed Overleaf template, available here:

`https://tinyurl.com/2400-sp25-pa1temp`

Please copy paste the content into your own Overleaf document to generate your report. This template gives you a feel for the type of content we expect in the following PAs. The templates for PA 2-5 will be much less detailed.

The quantitative evaluation section of your report must include a performance plot. The plot should have the input to `sqrt` on the x-axis and the wall-clock runtime on the y-axis. Plot a line for each of the three implementations of `sqrt` (iterative, recursive, sqrt from math.h). Ensure your plot is easy to read with a legend, reasonable font sizes, and appropriate colors/markers for black-and-white printing. You must discuss these results in the quantitative evaluation section.

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