ECE 2400 Computer Systems Programming Spring 2025

Topic 15: Trees

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Cornell University

revision: 2025-04-21-00-27

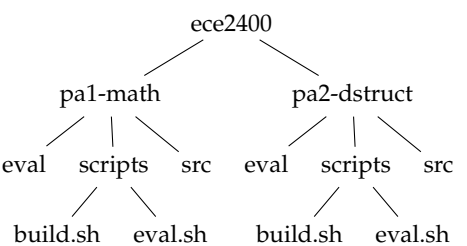
zyBooks logo indicates readings and coding labs in the course zyBook which will not be discussed in detail in lecture. Students are responsible for all material covered in lecture and in the course zyBook.

1. Tree Basics

Use Case 1: representing the Linux filesystem with a tree

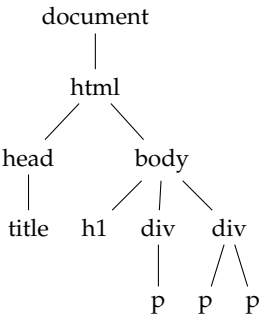
(No coincidence that the command is called `tree`!)

```
% tree ece2400
./ece2400
   |-- pa1-math
   |   |-- eval
   |   |-- scripts
   |   |   |-- build.sh
   |   |   |-- eval.sh
   |   |-- src
   |-- pa2-dstruct
       |-- eval
       |-- scripts
       |   |-- build.sh
       |   |-- eval.sh
       |-- src
```



Use Case 2: representing HTML/XML Document Object Model

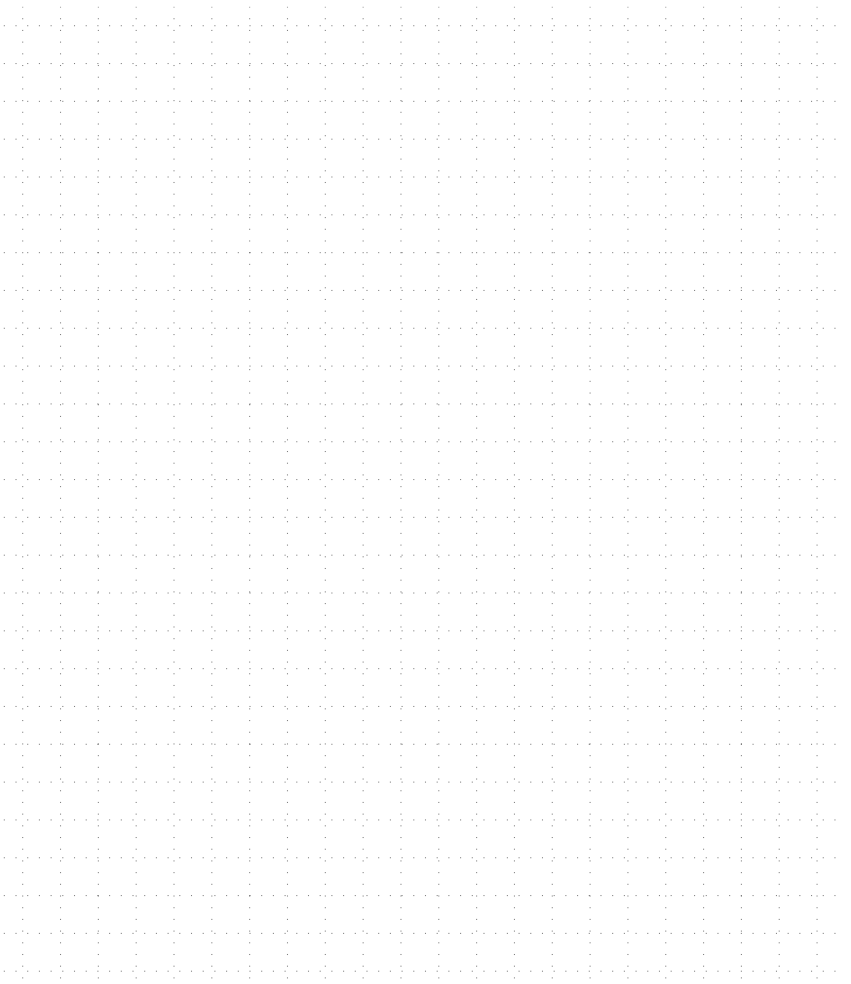
```
<html>
  <head>
    <title>Simple Website</title>
  </head>
  <body>
    <h1>Simple Website</h1>
    <div>
      <p>some content</p>
    </div>
    <div>
      <p>more content</p>
      <p>even more content</p>
    </div>
  </body>
</html>
```



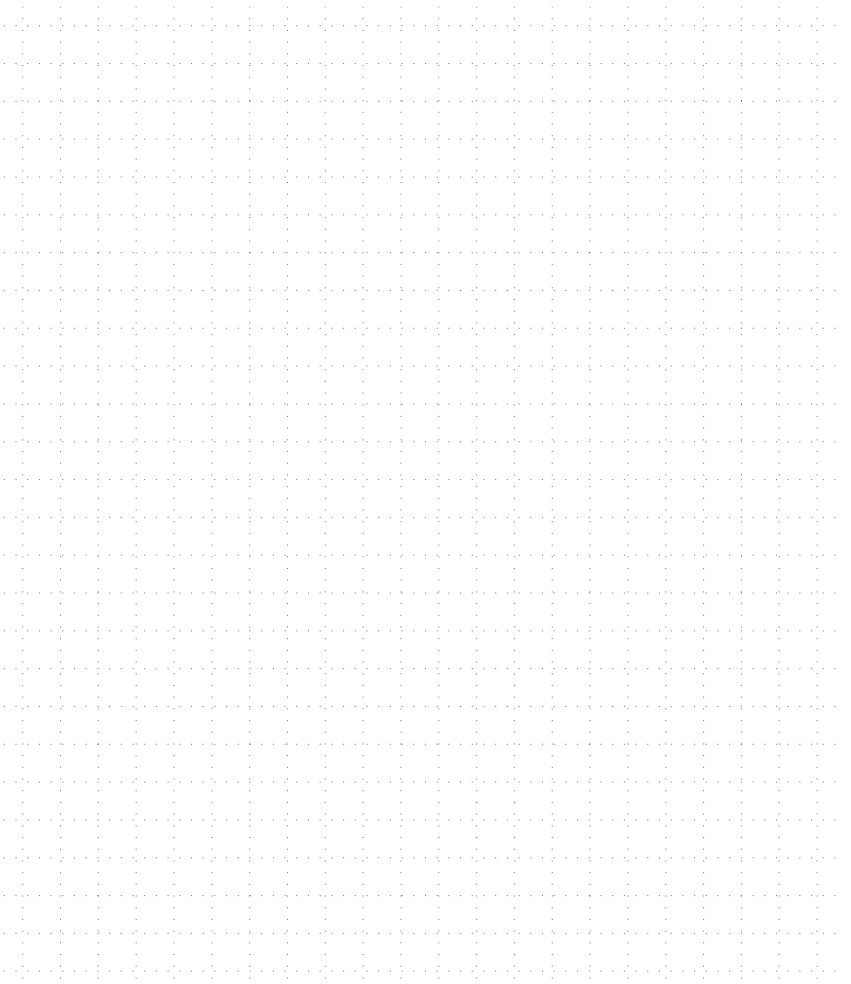
Trees can be ADTs with operations `insert`, `delete`,`find`, etc. However, in this class, we use **trees as efficient implementations of other ADTs**:

Recall from Topic 10:			**Implementation**			
ADT	List	Vector	Binary Search Tree	Binary Heap Tree	Lookup Table	Hash Table
Indexed Seq	✓	★				
Iterable Seq	★	★				
Stack	★	★				
Queue	★	★				
Priority Queue	✓	✓		★		
Set	✓	✓	★		★	★
Map	✓	✓	★		★	★

2. Tree Concepts



3. Tree Storage



4. Binary Trees

- Focus on object-oriented pointer-based binary tree storing `int`s
  - Could add iterators to improve data encapsulation
  - Could use object-oriented programming and dynamic polymorphism
  - Could use generic programming and static polymorphism
  - Later: could use concurrent programming to analyze tree in parallel

```
1  class BinaryTreeInt
2  {
3   public:
4
5    BinaryTreeInt();
6    ~BinaryTreeInt();
7
8    void insert_root( int v );
9    void insert_left( Node* node_p, int v );
10   void insert_right( Node* node_p, int v );
11
12   void print() const;
13
14   struct Node
15   {
16     Node( Node* p, int v );
17     int   value;
18     Node* parent_p;
19     Node* left_p;
20     Node* right_p;
21   };
22
23   Node* m_root_p;
24 };
```

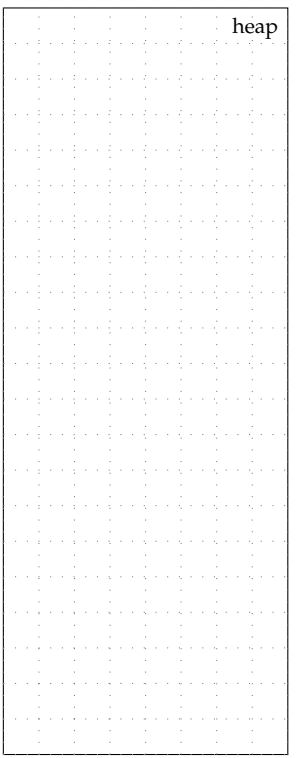
- Let's defer implementing `print` and destructor for now

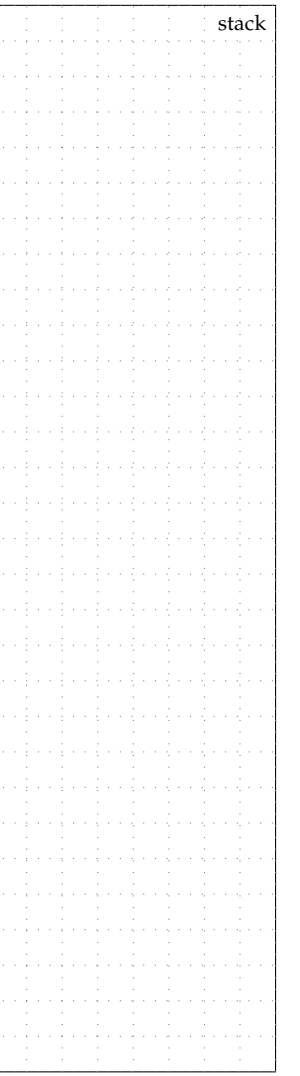
```
BinaryTreeInt::Node::Node( Node* p, int v )
{
  parent_p = p; value = v; left_p = nullptr; right_p = nullptr;
}

BinaryTreeInt::BinaryTreeInt()
{
  m_root_p = nullptr;
}

void BinaryTreeInt::insert_root( int v )
{
  m_root_p = new Node(nullptr,v);
}

void BinaryTreeInt::insert_left( Node* node_p, int v )
{
  node_p->left_p = new Node(node_p,v);
}

void BinaryTreeInt::insert_right( Node* node_p, int v )
{
  node_p->right_p = new Node(node_p,v);
}
```

Draw the tree resulting from this code sequence:

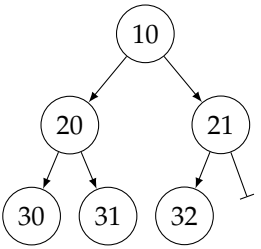
```
BinaryTreeInt bt;
bt.insert_root( 10 );
BinaryTreeInt::Node* r
 = bt.m_root_p;
bt.insert_left ( r, 11 );
bt.insert_right( r, 12 );
bt.insert_left ( r->left_p, 13 );
```

```
01 int main( void )
02 {
03   BinaryTreeInt bt;
04   bt.insert_root( 10 );
05   BinaryTreeInt::Node* r
06    = bt.m_root_p;
07   bt.insert_left ( r, 11 );
08   bt.insert_right( r, 12 );
09   bt.insert_left ( r->left_p, 13 );
10   return 0;
11 }
```





3 orderings for Tree Traversals



Traversal Task 1: Printing a tree (member function `print`)

When writing recursive helper function `print_h`, which traversals work?

```
void BinaryTreeInt::print() const {
  print_h( m_head_p );
}
void BinaryTreeInt::print_h( Node* node_p ) const {
```

[See zybook section 15.1]

Traversal Task 2: Deleting a tree (destructor)

When writing recursive helper function `destruct_h`, which traversals work?

```
BinaryTreeInt::~BinaryTreeInt() {
  destruct_h( m_head_p );
}
void BinaryTreeInt::destruct_h( Node* node_p ) {
```

5. Binary Search Trees

- Recall that sets provide `add` and `contains` member functions
- Recall that maps provide `add` and `lookup` member functions
- Consider implementing a set/map with a list or vector

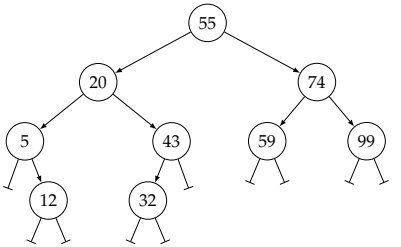
Time Complexity	`add` (*no duplicates: must do contains first!)*	`contains` / `lookup`
list		
list (sorted)		
vector		
vector (sorted)		
binary search tree		

- A binary search tree is a binary tree with the following invariant:

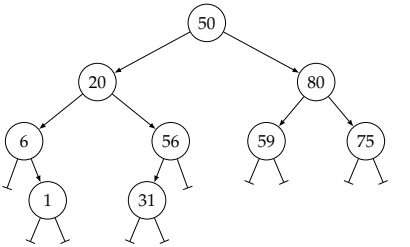
For any node in the tree with value v,
all values in the left subtree of that node are less than v and
all values in the right subtree of that node are greater than v.

- We can use a binary search tree to achieve $O(\log(N))$ time complexity for both `add` and `contains/lookup`
- This time complexity bound assumes binary tree is balanced which may or may not be a reasonable assumption

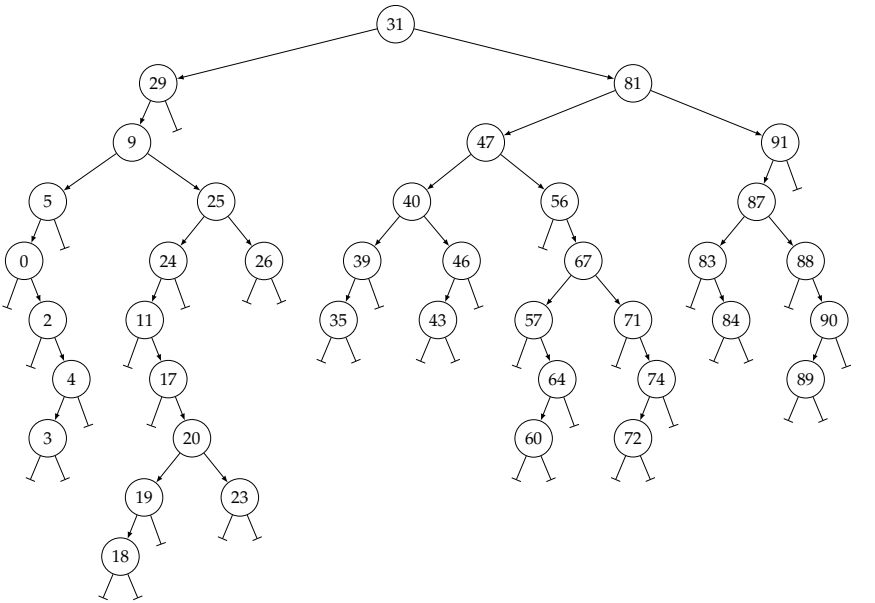
BST invariant is true



BST invariant is not true



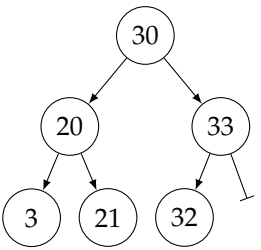
Larger BST



- Focus on object-oriented pointer-based binary search tree storing `ints` to implement a set
  - Could apply same approach to implementing a map
  - Could use object-oriented programming and dynamic polymorphism
  - Could use generic programming and static polymorphism
  - Later: could use concurrent programming to analyze tree in parallel

```
1  class BinarySearchTreeInt
2  {
3   public:
4    BinarySearchTreeInt();
5    ~BinarySearchTreeInt();
6
7    void add( int v );
8    bool contains( int v ) const;
9
10   private:
11
12   struct Node
13   {
14     Node( Node* p, int v );
15     int   value;
16     Node* parent_p;
17     Node* left_p;
18     Node* right_p;
19   };
20
21   void destruct_h( Node* node_p );
22   void add_h( Node* node_p, int v );
23   bool contains_h( Node* node_p, int v ) const;
24
25   Node* m_root_p;
26 };
```

Traversal Task 3: finding a value in a tree (member function `contains`)
When writing recursive helper function `contains_h`, which traversals work?




```
bool BinarySearchTreeInt::contains( int v ) const {
  return contains_h( m_root_p, v );
}

bool BinarySearchTreeInt::
  contains_h( Node* node_p, int v ) const {









}
```

Traversal Task 4: adding a value to a tree (member function `add`)

Version 1 of recursive helper function `add_h` is **clunky**.

```
1  void BinarySearchTreeInt::add( int v ) {
2    if ( m_root_p == nullptr ) {
3      m_root_p = new Node( nullptr, v );
4      return;
5    }
6
7    add_h( m_root_p, v );
8  }
9
10 void BinarySearchTreeInt::add_h( Node* node_p, int v )
11 {
12   assert( node_p != nullptr );
13
14   // base case: value is already in the tree
15   if ( v == node_p->value )
16     return;
17
18   // base case: add new node on right
19   if ( (v > node_p->value) && (node_p->right_p == nullptr) ) {
20     node_p->right_p = new Node( node_p, v );
21     return;
22   }
23
24   // base case: add new node on left
25   if ( (v < node_p->value) && (node_p->left_p == nullptr) ) {
26     node_p->left_p = new Node( node_p, v );
27     return;
28   }
29
30   // recursive case
31   if ( v > node_p->value )
32     add_h( node_p->right_p, v );
33   else
34     add_h( node_p->left_p, v );
35 }
```

Traversal Task 4: adding a value to a tree (member function `add`)

Version 2 of recursive helper function `add_h` is **elegant**!

```
1  void BinarySearchTreeInt::add(int v) {
2    m_steps = 0;
3    m_root_p = add_h(m_root_p, nullptr, v);
4  }
5
6  BinarySearchTreeInt::Node *
7   BinarySearchTreeInt::add_h(Node *node_p, Node *p, int v) {
8    m_steps++;
9
10   // base case: found place to insert new node
11   if (node_p == nullptr)
12     return new Node(p, v);
13
14   // base case: value is already in the tree
15   if (v == node_p->value)
16     return node_p;
17
18   // recursive case
19   if (v > node_p->value)
20     node_p->right_p = add_h(node_p->right_p, node_p, v);
21   else
22     node_p->left_p = add_h(node_p->left_p, node_p, v);
23
24   return node_p;
25 }
```

See zybook section 15.1 for complete runnable Versions 1 and 2!

Important: neither version handles the case where a new node needs to be created between an existing parent and child.