

ECE 2400 Computer Systems Programming

Spring 2025

Topic 15: Trees

School of Electrical and Computer Engineering
Cornell University

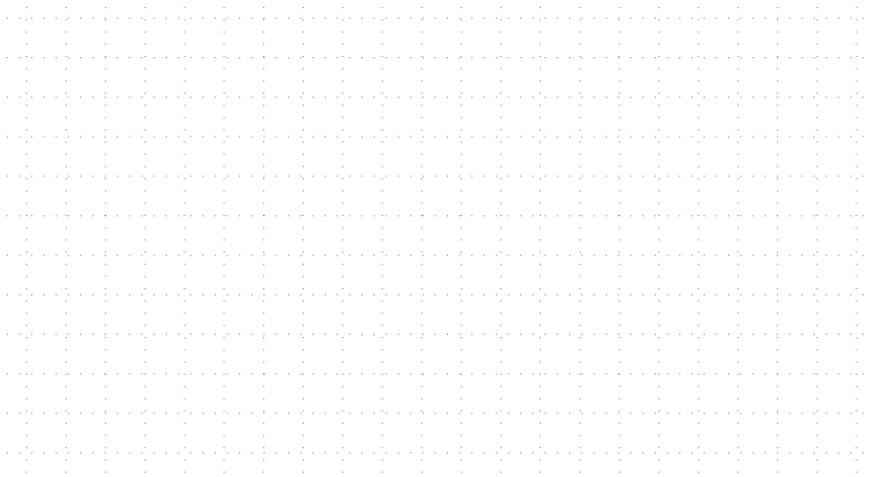
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zyBooks logo indicates readings and coding labs in the course zyBook which will not be discussed in detail in lecture. Students are responsible for all material covered in lecture and in the course zyBook.

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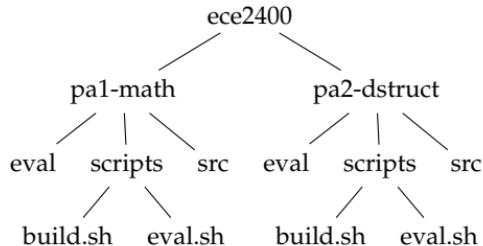
1. Tree Basics



Use Case 1: representing the Linux filesystem with a tree

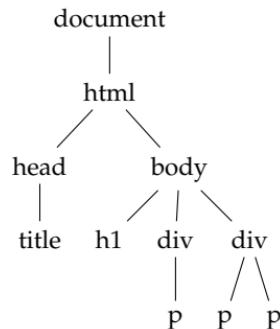
(No coincidence that the command is called *tree*!)

```
% tree ece2400
./ece2400
|-- pa1-math
|   |-- eval
|   |-- scripts
|   |   |-- build.sh
|   |   |-- eval.sh
|   |-- src
|-- pa2-dstruct
    |-- eval
    |-- scripts
    |   |-- build.sh
    |   |-- eval.sh
    |-- src
```



Use Case 2: representing HTML/XML Document Object Model

```
<html>
  <head>
    <title>Simple Website</title>
  </head>
  <body>
    <h1>Simple Website</h1>
    <div>
      <p>some content</p>
    </div>
    <div>
      <p>more content</p>
      <p>even more content</p>
    </div>
  </body>
</html>
```



Trees can be ADTs with operations `insert`, `delete`, `find`, etc. However, in this class, we use **trees as efficient implementations of other ADTs**:

Recall from Topic 10:

Implementation

ADT	List	Vector	Binary Search Tree	Binary Heap Tree	Lookup Table	Hash Table
Indexed Seq	✓	★				
Iterable Seq	★	★				
Stack	★	★				
Queue	★	★				
Priority Queue	✓	✓		★		
Set	✓	✓	★		★	★
Map	✓	✓	★		★	★

2. Tree Concepts

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	-----

3. Tree Storage

Storage Method	Structure	Advantages	Disadvantages
Adjacency List	Graph representation where each node is associated with a list of its adjacent nodes.	Efficient for sparse graphs and allows for easy insertion and deletion of edges.	Space complexity is proportional to the number of edges.
Adjacency Matrix	Graph representation where each node is represented by a row and a column, and the presence of an edge between two nodes is indicated by a value (e.g., 1 or true) in the corresponding matrix cell.	Space complexity is proportional to the number of nodes squared.	Space complexity is high for sparse graphs.
Degree Vector	A vector where each element represents the degree of a specific node in the graph.	Space complexity is proportional to the number of nodes.	Only provides information about the degree of each node, not the connections themselves.
Incidence Matrix	Graph representation where each node is represented by a row and each edge is represented by a column, indicating the incidence of edges at each node.	Space complexity is proportional to the number of nodes times the number of edges.	Only provides information about the edges and their incidence, not the connections between nodes.
Adjacency List with Node Data	Adjacency list structure where each node entry contains both the list of adjacent nodes and additional data specific to that node.	Space efficiency for graphs with many nodes and few edges.	May require more memory for node-specific data.
Adjacency Matrix with Node Data	Adjacency matrix structure where each node entry contains both the matrix cell value and additional data specific to that node.	Space efficiency for graphs with many nodes and few edges.	May require more memory for node-specific data.
Edge List	A list of edges, where each edge is represented by a pair of node indices.	Space efficiency for graphs with many edges and few nodes.	May require more memory for edge indices.
Edge Matrix	A matrix where each row and column index corresponds to a node, and the value at the intersection of a row and column indicates the presence of an edge between those nodes.	Space efficiency for graphs with many edges and few nodes.	May require more memory for edge indices.
Adjacency List with Edge Data	Adjacency list structure where each edge entry contains both the list of adjacent nodes and additional data specific to that edge.	Space efficiency for graphs with many edges and few nodes.	May require more memory for edge-specific data.
Adjacency Matrix with Edge Data	Adjacency matrix structure where each edge entry contains both the matrix cell value and additional data specific to that edge.	Space efficiency for graphs with many edges and few nodes.	May require more memory for edge-specific data.
Adjacency Matrix with Node and Edge Data	Adjacency matrix structure where each cell entry contains both node and edge-specific data.	Space efficiency for graphs with many edges and few nodes.	May require more memory for both node and edge-specific data.
Incidence Matrix with Node Data	Incidence matrix structure where each node entry contains both the matrix cell value and additional data specific to that node.	Space efficiency for graphs with many edges and few nodes.	May require more memory for node-specific data.
Incidence Matrix with Edge Data	Incidence matrix structure where each edge entry contains both the matrix cell value and additional data specific to that edge.	Space efficiency for graphs with many edges and few nodes.	May require more memory for edge-specific data.
Incidence Matrix with Node and Edge Data	Incidence matrix structure where each cell entry contains both node and edge-specific data.	Space efficiency for graphs with many edges and few nodes.	May require more memory for both node and edge-specific data.

4. Binary Trees

- Focus on object-oriented pointer-based binary tree storing ints
 - Could add iterators to improve data encapsulation
 - Could use object-oriented programming and dynamic polymorphism
 - Could use generic programming and static polymorphism
 - Later: could use concurrent programming to analyze tree in parallel

```
1  class BinaryTreeInt
2  {
3      public:
4
5      BinaryTreeInt();
6      ~BinaryTreeInt();
7
8      void insert_root( int v );
9      void insert_left( Node* node_p, int v );
10     void insert_right( Node* node_p, int v );
11
12    void print() const;
13
14    struct Node
15    {
16        Node( Node* p, int v );
17        int value;
18        Node* parent_p;
19        Node* left_p;
20        Node* right_p;
21    };
22
23    Node* m_root_p;
24 }
```

- Let's defer implementing print and destructor for now

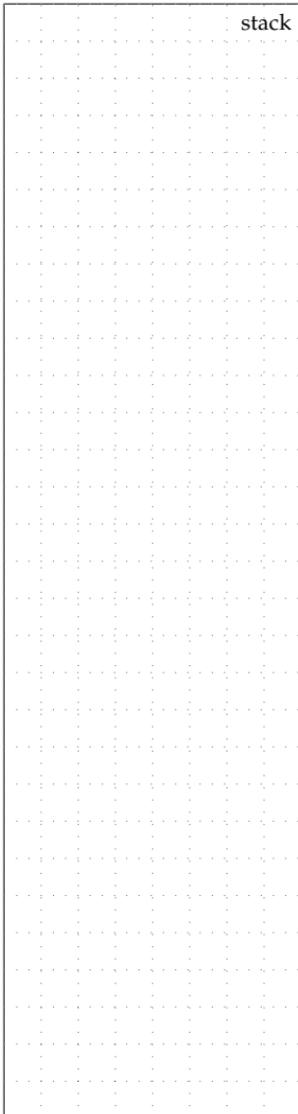
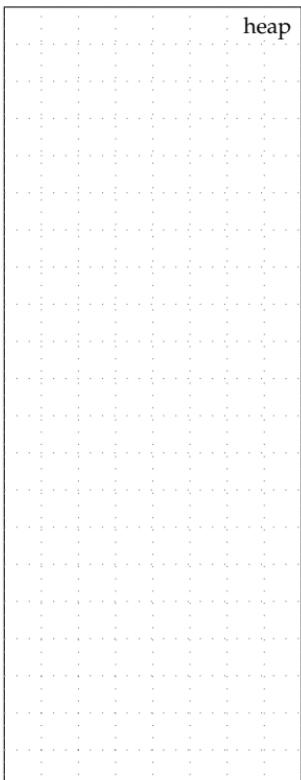
```
1  BinaryTreeInt::Node::Node( Node* p, int v )
2  {
3      parent_p = p; value = v; left_p = nullptr; right_p = nullptr;
4  }
5
6  BinaryTreeInt::BinaryTreeInt()
7  {
8      m_root_p = nullptr;
9  }
10
11 void BinaryTreeInt::insert_root( int v )
12 {
13     m_root_p = new Node(nullptr,v);
14 }
15
16 void BinaryTreeInt::insert_left( Node* node_p, int v )
17 {
18     node_p->left_p = new Node(node_p,v);
19 }
20
21 void BinaryTreeInt::insert_right( Node* node_p, int v )
22 {
23     node_p->right_p = new Node(node_p,v);
24 }
```

Draw the tree resulting
from this code sequence:

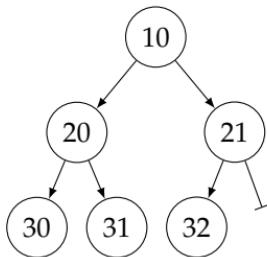
```
1  BinaryTreeInt bt;
2  bt.insert_root( 10 );
3  BinaryTreeInt::Node* r
4  = bt.m_root_p;
5  bt.insert_left ( r, 11 );
6  bt.insert_right( r, 12 );
7  bt.insert_left ( r->left_p, 13 );
```

4. Binary Trees

```
01 int main( void )
02 {
03     BinaryTreeInt bt;
04     bt.insert_root( 10 );
05     BinaryTreeInt::Node* r
06         = bt.m_root_p;
07     bt.insert_left ( r, 11 );
08     bt.insert_right( r, 12 );
09     bt.insert_left ( r->left_p, 13 );
10     return 0;
11 }
```



3 orderings for Tree Traversals



Traversal Task 1: Printing a tree (member function print)

When writing recursive helper function `print_h`, which traversals work?

```
void BinaryTreeInt::print() const {
    print_h( m_head_p );
}
void BinaryTreeInt::print_h( Node* node_p ) const {
```

[See zybook section 15.1]

Traversal Task 2: Deleting a tree (destructor)

When writing recursive helper function `destruct_h`, which traversals work?

```
BinaryTreeInt::~BinaryTreeInt() {
    destruct_h( m_head_p );
}
void BinaryTreeInt::destruct_h( Node* node_p ) {
```

5. Binary Search Trees

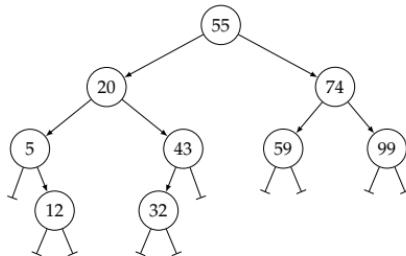
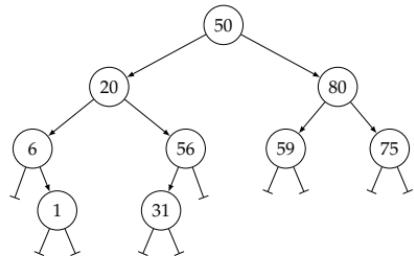
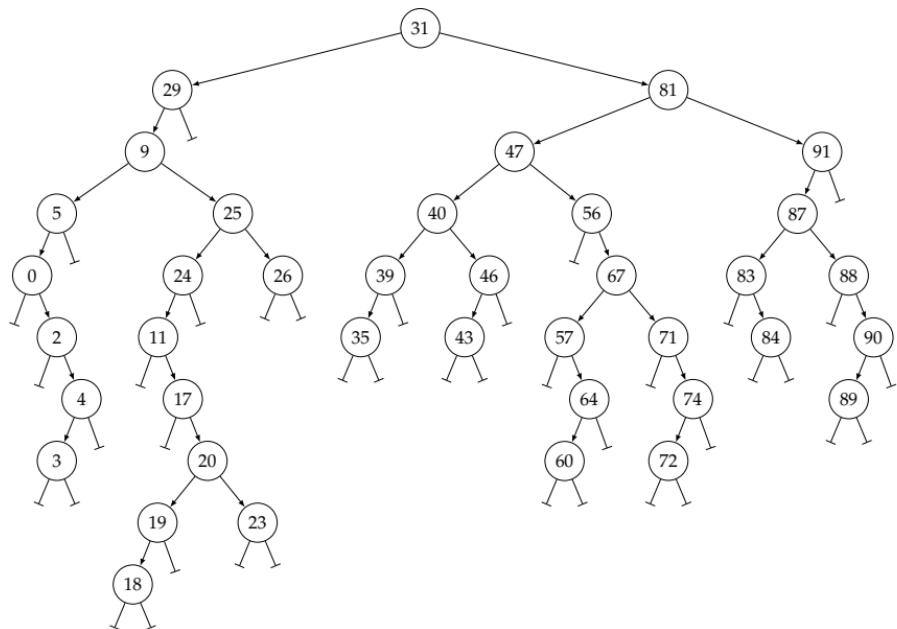
- Recall that sets provide add and contains member functions
- Recall that maps provide add and lookup member functions
- Consider implementing a set/map with a list or vector

Time Complexity	add (<i>no duplicates: must do contains first!</i>)	contains / lookup
list		
list (sorted)		
vector		
vector (sorted)		
binary search tree		

- A **binary search tree** is a binary tree with the following invariant:

For any node in the tree with value v ,
all values in the left subtree of that node are less than v and
all values in the right subtree of that node are greater than v .

- We can use a binary search tree to achieve $O(\log(N))$ time complexity for both add and contains/lookup
- This time complexity bound assumes binary tree is balanced which may or may not be a reasonable assumption

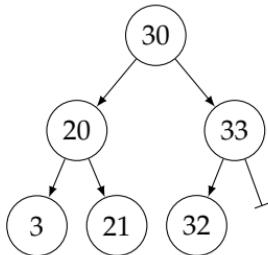
BST invariant is true**BST invariant is not true****Larger BST**

- Focus on object-oriented pointer-based binary search tree storing ints to implement a set
 - Could apply same approach to implementing a map
 - Could use object-oriented programming and dynamic polymorphism
 - Could use generic programming and static polymorphism
 - Later: could use concurrent programming to analyze tree in parallel

```
1  class BinarySearchTreeInt
2  {
3      public:
4          BinarySearchTreeInt();
5          ~BinarySearchTreeInt();
6
7          void add( int v );
8          bool contains( int v ) const;
9
10     private:
11
12     struct Node
13     {
14         Node( Node* p, int v );
15         int value;
16         Node* parent_p;
17         Node* left_p;
18         Node* right_p;
19     };
20
21     void destruct_h( Node* node_p );
22     void add_h( Node* node_p, int v );
23     bool contains_h( Node* node_p, int v ) const;
24
25     Node* m_root_p;
26 }
```

Traversal Task 3: finding a value in a tree (member function contains)

When writing recursive helper function `contains_h`, which traversals work?



```
bool BinarySearchTreeInt::contains( int v ) const {  
    return contains_h( m_root_p, v );  
}
```

```
bool BinarySearchTreeInt::  
contains_h( Node* node_p, int v ) const {
```

```
}
```

Traversal Task 4: adding a value to a tree (member function add)**Version 1** of recursive helper function add_h is **clunky**.

```
1 void BinarySearchTreeInt::add( int v ) {
2     if ( m_root_p == nullptr ) {
3         m_root_p = new Node( nullptr, v );
4         return;
5     }
6
7     add_h( m_root_p, v );
8 }
9
10 void BinarySearchTreeInt::add_h( Node* node_p, int v )
11 {
12     assert( node_p != nullptr );
13
14     // base case: value is already in the tree
15     if ( v == node_p->value )
16         return;
17
18     // base case: add new node on right
19     if ( (v > node_p->value) && (node_p->right_p == nullptr) ) {
20         node_p->right_p = new Node( node_p, v );
21         return;
22     }
23
24     // base case: add new node on left
25     if ( (v < node_p->value) && (node_p->left_p == nullptr) ) {
26         node_p->left_p = new Node( node_p, v );
27         return;
28     }
29
30     // recursive case
31     if ( v > node_p->value )
32         add_h( node_p->right_p, v );
33     else
34         add_h( node_p->left_p, v );
35 }
```

Traversal Task 4: adding a value to a tree (member function add)**Version 2** of recursive helper function add_h is elegant!

```
1 void BinarySearchTreeInt::add(int v) {  
2     m_steps = 0;  
3     m_root_p = add_h(m_root_p, nullptr, v);  
4 }  
5  
6 BinarySearchTreeInt::Node *  
7     BinarySearchTreeInt::add_h(Node *node_p, Node *p, int v) {  
8     m_steps++;  
9  
10    // base case: found place to insert new node  
11    if (node_p == nullptr)  
12        return new Node(p, v);  
13  
14    // base case: value is already in the tree  
15    if (v == node_p->value)  
16        return node_p;  
17  
18    // recursive case  
19    if (v > node_p->value)  
20        node_p->right_p = add_h(node_p->right_p, node_p, v);  
21    else  
22        node_p->left_p = add_h(node_p->left_p, node_p, v);  
23  
24    return node_p;  
25 }
```

See zybook section 15.1 for complete runnable Versions 1 and 2!

Important: neither version handles the case where a new node needs to be created between an existing parent and child.