# ECE 2400 Computer Systems Programming
## Fall 2019
### Topic 2: C Recursion

School of Electrical and Computer Engineering  
Cornell University  

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Sections marked with a star (✱) are not covered in lecture but are instead covered in the online lecture notes. Students are responsible for all material covered in lecture and in the online lecture notes. Material from the online lecture notes will definitely be assessed in the prelim and final exam.

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Our goal is to understand what the word “recursion” means, so we look up “recursion” in the dictionary ...

- Recursion is when the algorithm is defined in terms of itself
- No new syntax or semantics
- Understanding recursion simply involves applying what we have already learned with respect to functions, conditionals, iteration
1. Single Recursion

Recall from mathematics, the factorial of a number (n!) is:

\[
\begin{align*}
n! & = \begin{cases} 
1 & \text{if } n = 0 \\
n \times (n-1)! & \text{if } n > 0
\end{cases}
\end{align*}
\]

So in other words:

<table>
<thead>
<tr>
<th>n!</th>
<th>Calculation</th>
<th>Value</th>
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<tbody>
<tr>
<td>0!</td>
<td>=</td>
<td>1</td>
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<tr>
<td>1!</td>
<td>=</td>
<td>1</td>
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<tr>
<td>2!</td>
<td>= 1 \times 2</td>
<td>2</td>
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<tr>
<td>3!</td>
<td>= 1 \times 2 \times 3</td>
<td>6</td>
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<tr>
<td>4!</td>
<td>= 1 \times 2 \times 3 \times 4</td>
<td>24</td>
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<td>5!</td>
<td>= 1 \times 2 \times 3 \times 4 \times 5</td>
<td>120</td>
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We can write a function to calculate factorial using a for loop:

```c
int factorial( int n ) {
    int result = 1;
    for ( int i = 1; i <= n; i++ )
        result = result * i;
    return result;
}
```
1. Single Recursion

• The loop implementation does not really resemble the original mathematical formulation

• The mathematical formulation is inherently recursive

• Can we implement factorial more directly using recursion?

\[
n! = \begin{cases} 
1 & \text{if } n = 0 \\
n \times (n - 1)! & \text{if } n > 0 
\end{cases}
\]
1. Single Recursion

We can use the exact same “by-hand” execution approach we learned in the previous topic to understand recursion.

```c
int factorial( int n )
{
    // base case
    if ( n == 0 ) {
        return 1;
    }
    // recursive case
    if ( n > 0 ) {
        return n * factorial(n-1);
    }
}
int main()
{
    int a = factorial(3);
    return 0;
}
```

Questions:

- What if n is negative?
- What if the execution arrow reaches end of a non-void function without encountering a return statement?
2. Multiple Recursion

Recall from mathematics, the Fibonacci sequence is a sequence of integers such that every number after the first two is the sum of the two preceding ones:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

The numbers in the Fibonacci sequence are called “Fibonacci numbers”. By definition, the first two numbers in the Fibonacci sequence are 0 and 1. Ancient scholars realized the importance of this sequence in both mathematics and nature. Fibonacci sequences can be found in the arrangement of leaves on a stem or pattern in a pine cone.

We can write a function to calculate the \( n \)th Fibonacci number using a for loop:

```c
int fib( int n ) {
    // by definition
    if (n == 0) return 0;
    if (n == 1) return 1;

    int fib_minus2 = 0;
    int fib_minus1 = 1;
    int result = 0;

    for ( int i=2; i<=n; i++ ) {
        result = fib_minus1
                  + fib_minus2;
        fib_minus2 = fib_minus1;
        fib_minus1 = result;
    }
    return result;
}
```
Can we implement factorial more elegantly using recursion?

Illustrating call tree for fib
3. Writing a Recursive Function

Write pseudo-code for a recursive function which draws the tick marks on a vertical ruler. The middle tick mark should be the longest and mark the 1/2 way point, slightly shorter tick marks should mark the 1/4 way points, even slightly shorter tick marks should mark the 1/8 way points and so on. The function should take one argument: the height of the middle tick mark (i.e., the number of dashes). The function should always return 0. Use printf to display the tick marks.

```
int ruler( int height ) {

}
```
3. Writing a Recursive Function

<table>
<thead>
<tr>
<th>ruler(1)</th>
<th>ruler(2)</th>
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- Step 1: Work an example yourself
  - height = 2, height = 3

- Step 2: Write down what you just did
  - What is the base case?
  - What is the recursive case?

- Step 3: Generalize your steps
  - for any height

- Step 4: Test your algorithm
  - does it work for height = 4?

- Step 5: Translate to pseudocode
3. Writing a Recursive Function

Think about the recursive call tree?

Manually work through example ruler