# ECE 2400 Computer Systems Programming Fall 2021 <br> Topic 2: C Recursion 

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1 Single Recursion ..... 3
2 Multiple Recursion ..... 6
3 Writing a Recursive Function ..... 8
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Our goal is to understand what the word "recursion" means, so let's look up "recursion" in the dictionary ...

```
                                    Dictionary (2 found)
< A A
All Dictionary Thesaurus Apple Wikipedia
recursion recursion | ra'kərZHən |
    recursion for..
        noun Mathematics & Linguistics
        the repeated application of a recursive procedure or definition.
        - a recursive definition.
    ORIGIN
```

        1930s: from late Latin recursio( \(n\)-), from recurrere 'run back' (see RECUR) .
    - Recursion is when the algorithm is defined in terms of itself
- No new syntax or semantics
- Understanding recursion simply involves applying what we have already learned with respect to functions, conditionals, iteration


## 1. Single Recursion

Recall from mathematics, the factorial of a number ( $n!$ ) is:

$$
n!= \begin{cases}1 & \text { if } n=0 \\ n \times(n-1)! & \text { if } n>0\end{cases}
$$

So in other words:

| $0!=$ | $=1$ |
| :--- | :--- |
| $1!=$ | $=1$ |
| $2!=1 \times 2$ | $=2$ |
| $3!=1 \times 2 \times 3$ | $=6$ |
| $4!=1 \times 2 \times 3 \times 4$ | $=24$ |
| $5!=1 \times 2 \times 3 \times 4 \times 5=120$ |  |

We can write a function to calculate factorial using a for loop:

```
1 int factorial( int n ) {
2 int result = 1;
3 for ( int i = 1; i <= n; i++ )
4 result = result * i;
5 return result;
6 }
```

- The loop implementation does not really resemble the original mathematical formulation
- The mathematical formulation is inherently recursive
- Can we implement factorial more directly using recursion?

$$
n!= \begin{cases}1 & \text { if } n=0 \\ n \times(n-1)! & \text { if } n>0\end{cases}
$$

We can use the exact same "by-hand" execution approach we learned in the previous topic to understand recursion.

## Questions:

- What if n is negative?
- What if the execution arrow reaches end of a non-void function without encountering a return statement?

```
int factorial( int n )
```

int factorial( int n )
{
{
// base case
// base case
if ( n == 0 ) {
if ( n == 0 ) {
return 1;
return 1;
}
}
// recursive case
// recursive case
if ( n > 0 ) {
if ( n > 0 ) {
return n *
return n *
factorial(n-1);
factorial(n-1);
}
}
}
}
int main()
int main()
{
{
int a = factorial(3);
int a = factorial(3);
return 0;
return 0;
}

```
}
```


## 2. Multiple Recursion

Recall from mathematics, the Fibonacci sequence is a sequence of integers such that every number after the first two is the sum of the two preceding ones:

$$
0,1,1,2,3,5,8,13,21,34,55,89,144, \ldots
$$

The numbers in the Fibonacci sequence are called "Fibonacci numbers". By definition, the first two numbers in the Fibonacci sequence are 0 and 1. Ancient scholars realized the importance of this sequence in both mathematics and nature. Fibonacci sequences can be found in the arrangement of leaves on a stem or patterns in a pine cone.

We can write a function to calculate the $\mathrm{n}^{\text {th }}$ Fibonacci number using a for loop:

```
int fib( int n ) {
    // by definition
    if (n == 0) return 0;
    if (n == 1) return 1;
    int fib_minus2 = 0;
    int fib_minus1 = 1;
    int result = 0;
    for ( int i=2; i<=n; i++ ) {
        result = fib_minus1
        + fib_minus2;
        fib_minus2 = fib_minus1;
        fib_minus1 = result;
    }
    return result;
}
```

Can we implement factorial more elegantly using recursion?

## Illustrating call tree for fib

## 3. Writing a Recursive Function

Write pseudo-code for a recursive function which draws the tick marks on a vertical ruler. The middle tick mark should be the longest and mark the $1 / 2$ way point, slightly shorter tick marks should mark the $1 / 4$ way points, even slightly shorter tick marks should mark the $1 / 8$ way points and so on. The function should take one argument: the height of the middle tick mark (i.e., the number of dashes). The function should always return 0 .

ruler(1) ruler(2) ruler(3) ruler(4) ruler(5)

- Step 1: Work an example yourself
- height $=2$, height $=3$
- Step 2: Write down what you just did
- What is the base case?
- What is the recursive case?
- Step 3: Generalize your steps
- for any height
- Step 4: Test your algorithm
- does it work for height $=4$ ?
- Step 5: Translate to pseudocode


## Think about the recursive call tree?

## Manually work through example ruler

