ECE 2400 Computer Systems Programming, Fall 2017
Programming Assignment 1: Math Functions

School of Electrical and Computer Engineering
Cornell University

revision: 2017-09-19-14-22

1. Introduction

The first programming assignment is a warmup assignment done individually that is designed to give you experience with two important aspects of computer systems programming: software design and software verification. In this assignment, you will leverage the basic concepts from lecture, ranging from variables and operators to conditional and iteration statements. More advanced concepts such as recursion will also play a key role in optimizing the performance of your code.

You will design and implement the *power* and *square root* math functions twice: first using a simple but naive implementation, and then using a more sophisticated algorithm that can potentially improve performance by an order of magnitude. Throughout the assignment, you will carefully design your code to be *maintainable* ("Will somebody else be able to understand my code and extend it?"). We will conduct individual code reviews via GitHub to help you improve the quality of the code you write. You will also use unit testing to carefully verify that your design is *robust* ("Will my code handle all situations as intended?"). Just as in the tutorial, we will leverage the Criterion framework for unit testing, TravisCI for continuous integration testing, and Codecov.io for code coverage analysis.

After your code is functional and verified, you will write a 2–4 page design report that describes your design, discusses your testing strategy, and evaluates the performance (and other trade-offs) of the baseline and alternative implementations. **While the final code and report are all due at the end of the assignment, we also require meeting an incremental milestone of pushing your baseline implementations to GitHub on the date specified by the instructors**.

This handout assumes that you have read and understand the course tutorials. To get started, log in to an `ecelinux` machine, source the setup script, and clone your individual remote repository from GitHub:

```
% source setup-ece2400.sh
% mkdir -p ${HOME}/ece2400
% cd ${HOME}/ece2400
% git clone git@github.com:cornell-ece2400/<netid>
```

Where `<netid>` should be replaced with your NetID. **You should never fork your individual remote repository! If you need to work in isolation then use a branch within your individual remote repository.** If you have already cloned your individual remote repository, then use `git pull` to ensure you have any recent updates before running all of the tests. You can run all of the tests in the lab like this:

```
% cd ${HOME}/ece2400/<netid>
% git pull --rebase
% mkdir -p pa1-math/build
% cd pa1-math/build
% ../configure
```

```
% make check
```

All of the tests should fail since you have not implemented the programming assignment yet. For this assignment, you will work in the `pa1-math` subproject, which includes the following files:

- `configure` – Configuration script to generate Makefile
- `Makefile.in` – Makefile template to run tests and eval
- `src/pow-v1.c` – Source code for baseline `pow`
- `src/pow-v1.h` – Header file for baseline `pow`
- `src/pow-v1-test.c` – Test cases for baseline `pow`
- `src/pow-v1-eval.c` – Evaluation program for baseline `pow`
- `src/pow-v2.c` – Source code for alternative `pow`
- `src/pow-v2.h` – Header file for alternative `pow`
- `src/pow-v2-test.c` – Test cases for alternative `pow`
- `src/pow-v2-eval.c` – Evaluation program for alternative `pow`
- `src/sqrt-v1.c` – Source code for baseline `sqrt`
- `src/sqrt-v1.h` – Header file for baseline `sqrt`
- `src/sqrt-v1-test.c` – Test cases for baseline `sqrt`
- `src/sqrt-v1-eval.c` – Evaluation program for baseline `sqrt`
- `src/sqrt-v2.c` – Source code for alternative `sqrt`
- `src/sqrt-v2.h` – Header file for alternative `sqrt`
- `src/sqrt-v2-test.c` – Test cases for alternative `sqrt`
- `src/sqrt-v2-eval.c` – Evaluation program for alternative `sqrt`
- `src/pow.dat` – Input dataset for `pow` evaluation
- `src/sqrt.dat` – Input dataset for `sqrt` evaluation
- `scripts` – Scripts for the build system and generating datasets

2. Design Description: Baseline Implementation

In this section, you will be implementing a simple but naive algorithm to compute the *power* and *square root* math functions. The name of our power function will be `pow`, and the name of our square root function will be `sqrt`.

The `pow` function takes two input arguments: (1) the base value (b), and (2) the exponent value (e). The function returns the base raised to the power of the exponent (i.e., b^e). Specifically, the corresponding C function has the following function signature:

```
double pow( double base, int exponent )
```

Notice that *base* is a floating-point double (i.e., a real number), *exponent* is an integer, and the returned value is also a floating-point double. A basic implementation of the `pow` function expressed in a mathematical formula is:

$$b^e = \begin{cases} 1 & \text{if } e = 0 \\ b \times b \times \overset{e}{\cdots} \times b & \text{if } e > 0 \\ 1/(b \times b \times \overset{|e|}{\cdots} \times b) & \text{if } e < 0 \end{cases}$$

Your baseline implementation of the `pow` function should directly correspond to the above definition and should use an iteration statement with a few carefully chosen conditional statements.

In this assignment, we will allow certain cases in the implementation of `pow` to remain undefined:

- A base of 0 raised to a negative exponent is undefined
- A base of 0 raised to the 0^{th} power is undefined
- Any cases involving overflowing a double are undefined

We will not test these cases when we grade the assignment, and students do not need to handle these special cases in their implementation. Write your baseline implementation for `pow` inside of `src/pow-v1.c`.

The `sqrt` function takes one input argument x and returns its square root (i.e., $\sqrt{x}$). The corresponding C function has the following function signature:

```
int sqrt( int x )
```

Notice that the variant of `sqrt` that we will use in this assignment takes an integer input and returns another integer. The return value is the square root of x rounded down to the nearest integer. For example, calling `sqrt(5)` will return 2. If x is a negative value, the `sqrt` function must return -1 to report an invalid input.

The baseline implementation of the `sqrt` function should be implemented using an iteration statement. Let i range from zero to x. For each i, compute $i \times i$ and compare the result with x. If $i \times i$ is smaller than x, then i is less than the square root of x. If $i \times i$ is larger than x, then i is greater than the square root of x. By gradually testing all values of i, you will be able to find the square root of x rounded down to the nearest integer. Write your baseline implementation for `sqrt` inside of `src/sqrt-v1.c`.

3. Design Description: Alternative Implementation

The algorithms used for the baseline implementations of `pow` and `sqrt` are simple but slow. In this section, you will implement a more sophisticated algorithm for each of the two math functions, both of which can potentially improve performance by an order of magnitude compared to their baseline versions.

The baseline implementation of `pow` is particularly slow when the exponent is large because: (1) the computer executes multiplication operations more slowly compared to simpler operations (e.g., addition, subtraction); and (2) the number of multiply operations increases linearly with e. We can exploit structure in the baseline implementation to reduce the required number of multiply operations. Consider the following example.

$$3^8 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

We can reduce the number of multiply operations by reusing intermediate results.

$$a = 3 \times 3$$
$$b = a \times a$$
$$c = b \times b$$

We first calculate $3^2 = 3 \times 3$, and we can then reuse 3^2 to calculate $3^4 = (3^2)^2$, and we can then reuse 3^4 to calculate $3^8 = ((3^2)^2)^2$. The baseline implementation would require seven multiply operations, while the alternative implementation requires only three multiply operations. Here is the alternative implementation expressed as a mathematical formula:

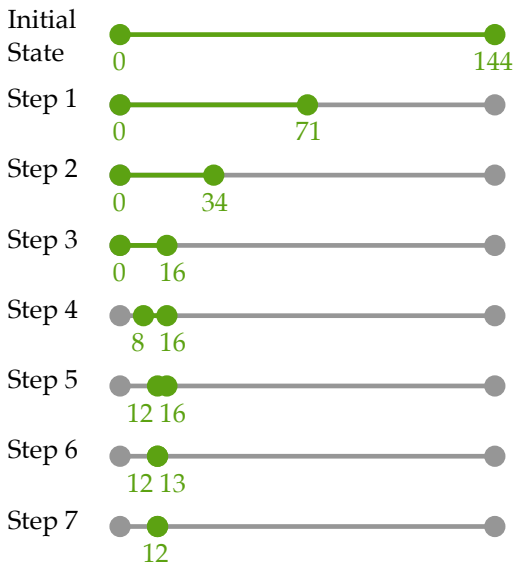
$$b^e = \begin{cases} 1 & \text{if } e = 0 \\ (b^2)^{e/2} & \text{if } e > 0 \text{ and } e \text{ is even} \\ b \times b^{e-1} & \text{if } e > 0 \text{ and } e \text{ is odd} \\ 1/(b^{|e|}) & \text{if } e < 0 \end{cases}$$

Notice that this approach for computing `pow` has multiple recursive cases. Although an iterative solution using loops is possible, we can more elegantly and concisely capture this algorithm using a recursive helper function. Write your alternative implementation for `pow` inside of `src/pow-v2.c`. You may add additional helper functions inside this file as needed.

The baseline implementation of `sqrt` is particularly slow when x is large because: (1) as mentioned above, the computer executes multiplication operations more slowly compared to simpler operations (e.g., addition, subtraction); and (2) the number of multiply operations increases linearly with x since we are doing an exhaustive search. We can use a more sophisticated search to reduce the number of multiply operations. Consider the situation when x is 144. We can divide the search space into two ranges:

- Range of integers from 0 to $(\frac{x}{2} - 1)$, which is $[0, 71]$ when x is 144
- Range of integers from $\frac{x}{2}$ to x, which is $[72, 144]$ when x is 144

We can quickly determine which half the square root of x lies in by squaring the midpoint (i.e., $72 \times 72 = 5184$) and comparing to x. Observing $5184 > 144$ tells us that our guess of 72 was much too high, so the answer must be in the lower half (i.e., somewhere in the range $[0, 71]$), which is true since we know in this example that the square root is 12. We can continue applying the same approach on the smaller range, dividing the search space into smaller and smaller ranges. Figure 1 illustrates an example execution when x is 144. This approach allows us to quickly "zero in" on the square root of x. We can capture this algorithm iteratively, but a recursive solution is also possible



- Split the range $[0, 144]$ into $[0, 71]$ and $[72, 144]$
- Choose the lower range
- Split the range $[0, 71]$ into $[0, 34]$ and $[35, 71]$
- Choose the lower range
- Split the range $[0, 34]$ into $[0, 16]$ and $[17, 34]$
- Choose the lower range
- Split the range $[0, 16]$ into $[0, 7]$ and $[8, 16]$
- Choose the upper range
- Split the range $[8, 16]$ into $[8, 11]$ and $[12, 16]$
- Choose the upper range
- Split the range $[12, 16]$ into $[12, 13]$ and $[14, 16]$
- Choose the lower range
- Search the range $[12, 13]$
- Identify the square root of 144 to be 12

Figure 1: Example of Alternative `sqrt` Algorithm – The range of integers that can contain the square root is halved at each step.

and may be more elegant and concise. The general approach of repeatedly halving the search space is known as a *binary search*. We will learn more about this class of algorithms in the future. Write your alternative implementation for `sqrt` inside of `src/sqrt-v2.c`. You may add additional helper functions inside this file as needed.

When squaring large integers, it is possible to exceed the maximum value an integer type can hold. This is called *integer overflow* and can corrupt the data in your program. This won't happen in the baseline implementation since we gradually increase the values we are squaring until we reach x. The alternative implementation can easily square larger integers as it uses the binary search to narrow the search space. It is relatively straight-forward to detect overflow before it happens. We have provided a short helper function named `detect_overflow_before_squaring` that takes an integer and returns 1 if x^2 will cause overflow and returns 0 if it will not. Please use this helper function before squaring integers to avoid corrupting the data in your program.

While you are required to implement the alternative implementations described in this section, students should also feel free to experiment with additional implementations. These implementations should be kept separate by using `pow-v3` and `sqrt-v3` prefixes. Students will have to modify the `Makefile` accordingly, and they will also need to ensure that any additional implementations are both tested and evaluated.

4. Testing Strategy

You are responsible for developing an effective testing strategy to ensure all implementations are correct. We provide one or two basic directed tests for each function, but you will need to add more directed test and random tests. Writing tests is one of the most important and challenging aspects of software programming. Designers often spend far more time designing tests than they do designing the actual program.

As you design your implementations, pay careful attention to corner cases and unexpected inputs (e.g., negative inputs) that break the functionality of your code. When you encounter such a case, capture the situation with a test case and a helpful error message and improve your code. Carefully read the design specification (i.e., the inputs, the outputs, and the behavior), so you know how your program should respond in all possible scenarios. Convince yourself that your implementations are *robust* by carefully developing a testing *strategy*.

In addition to writing directed tests, you should also add random tests. You can randomly generate inputs using the `rand` function in the standard C library (include `stdlib.h`). Use the `srand` function to initialize the random seed to a deterministic value to ensure your random tests are repeatable. You can use the `pow` and `sqrt` functions in the standard C library (include `math.h`) as golden reference models to generate correct reference outputs which you can then compare to the results from your own implementations. Note that you are not allowed to use the `pow` and `sqrt` functions in the standarc C library for your implementation, only for verification.

When testing the `pow` function, note that the result can vary by very small amounts because of precision errors that build up as the computer performs arithmetic on real numbers. This is commonly solved by comparing relative amounts (i.e., checking that the two numbers are within at least 99.99% of each other). An example test written in this fashion has been provided for you in `src/pow-v1-test.c` and `src/pow-v2-test.c`. If precision error becomes a problem, please compare numbers relatively.

5. Evaluation

Once you have verified the functionality of the baseline and alternative implementations, you can then start to evaluate the performance of these implementations. We provide you a performance analysis harness for each implementation. You can build and run these evaluation programs like this:

```
% cd ${HOME}/ece2400/<netid>
% mkdir -p pa1-math/build-eval
% cd pa1-math/build-eval
% ../configure --enable-eval
% make pow-v1-eval  && ./pow-v1-eval
% make sqrt-v1-eval && ./sqrt-v1-eval
% make pow-v2-eval  && ./pow-v2-eval
% make sqrt-v2-eval && ./sqrt-v2-eval
```

Note how we are working in a separate `build-eval` build directory, and that we are using the `--enable-eval` command line option to the `configure` script. This tells the build system to create optimized executables without any extra debugging information. You can build and run all of the evaluation programs in a single step like this:

```
% cd ${HOME}/ece2400/<netid>/pa1-math/build-eval
% make eval
```

The evaluation programs apply your math functions to 1000 inputs and report the total wall-clock run time. This will enable you to compare the performance between the baseline and alternative algorithms. The evaluation programs also ensure that your implemenations are producing the correct results, however, you should not use the evaluation programs for testing. If your implementations fail during the evaluation, then your testing strategy is insufficient. You must add more unit tests to effectively test your program before returning to performance evaluation.

Acknowledgments

This programming assignment was created by Christopher Batten, Jose Martínéz, Christopher Torng, Xiaodong Wang, Shuang Chen, and Shunning Jiang as part of the ECE 2400 Computer Systems Programming course at Cornell University.