ECE 2400 Computer Systems Programming
Fall 2017

Topic 18: Tables

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1. Tables Concepts

- Tables have rows and columns
- Rows are usually identified by a name (number or even a string)
- Columns are usually identified by a name (number or even a string)
- Rows may have multiple columns of data
- Tables are indexed to access row data at a certain column

	A	**B**	**C**
0			
1			
2			

Table as a Set ADT

Table as a Map ADT

2. Lookup Tables

- A lookup table is a table in which all potential data is known
- A lookup table is accessed by simple array indexing in O(1) time
- Lookup tables may be multidimensional

2.1. Set of Integers as a Lookup Table

- A set of unsigned integers in the range [0, 4]
- Implemented internally as a simple array of booleans

Index	**0**	**1**	**2**	**3**	**4**
Present?					

Interface

```
class LookupTableInt
{
 public:
  LookupTableInt();

  void add      ( unsigned int v );
  void remove   ( unsigned int v );
  bool contains ( unsigned int v );

 private:
  static const unsigned int size = 5;
  bool m_table[size];
};
```

Implementation

```
LookupTableInt::LookupTableInt()
{ }

void LookupTableInt::add( unsigned int v )
{
  m_table[v] = true;
}

void LookupTableInt::remove( unsigned int v )
{
  m_table[v] = false;
}

bool LookupTableInt::contains( unsigned int v )
{
  return m_table[v];
}
```

Main

```
int main( void )
{
  LookupTableInt table;

  table.add(3);     // Add value 3
  table.remove(3); // Remove value 3

  return 0;
}
```

2.2. Set of Strings as a Lookup Table

- A set of strings limited to "Apple", "Banana", "Cherry"
- Implemented internally as a simple array of booleans
- Must correspond each possible string to an internal array index

Index	**Apple**	**Banana**	**Cherry**
Present?			

Interface

```
class LookupTableString
{
 public:
  LookupTableString();

  void add      ( std::string v );
  void remove   ( std::string v );
  bool contains ( std::string v );

 private:
  unsigned int ind( std::string v );
  static const unsigned int size = 3;
  bool m_table[size];
};
```

Implementation

```
LookupTableString::LookupTableString()
{ }
```

```
void LookupTableString::add( std::string v )
{
  m_table[ ind(v) ] = true;
}

void LookupTableString::remove( std::string v )
{
  m_table[ ind(v) ] = false;
}

bool LookupTableString::contains( std::string v )
{
  return m_table[ ind(v) ];
}

unsigned int LookupTableString::ind( std::string v )
{
  if      ( v == "Apple"  ) return 0;
  else if ( v == "Banana" ) return 1;
  else if ( v == "Cherry" ) return 2;
  else throw std::out_of_range( "No such fruit!" );
}
```

Main

```
int main( void )
{
  LookupTableString table;

  table.add( "Cherry" );    // Add value "Cherry"
  table.remove( "Cherry" ); // Remove value "Cherry"

  return 0;
}
```

2.3. Strengths and Weaknesses of Lookup Tables

- We have implemented a set of integers as a lookup table
- We have implemented a set of strings as a lookup table
- Recall that lists, vectors, and even trees can also implement sets!

Set ADT as a List

Set ADT as a Tree

	`add`	`remove`	`contains`
list/vector	O(N)	O(N)	O(N)
tree	O(log(N))	O(log(N))	O(log(N))
lut	O(1)	O(1)	O(1)

Strengths of Lookup Tables

- Lookup tables have great time complexity!

Weaknesses of Lookup Tables

- I want a set for a larger range of numbers (e.g., from 0 to 1,000,000)
  - Lookup tables are ____________________
- I want a set of other types (e.g., strings)
  - Lookup tables are ____________________

2.4. Mitigating the Weaknesses of Lookup Tables

- Lookup tables have space complexity that is linear with k, the space of all possible unique inputs.

k = 4 and N = 2

k = 1,000,000 and N = 2

- How can we make the space complexity instead scale with N, the number of unique elements currently in the set?

Reduction Operations

- Define a `reduce` function that converts a large number into the range accessible by the internal array.
- When the internal array fills up, double the size of the array.

Member function to reduce a large number

```
unsigned int reduce( unsigned int v ) {
```

Interface

```
class LookupTableIntOneMillion
{
 public:
  LookupTableIntOneMillion();

  void add      ( unsigned int v );
  void remove   ( unsigned int v );
  bool contains ( unsigned int v );

 private:
  unsigned int reduce ( unsigned int v );
  static const unsigned int size = 10;
  bool m_table[size];
};
```

Implementation

```
LookupTableIntOneMillion::LookupTableIntOneMillion()
{ }

void LookupTableIntOneMillion::add( unsigned int v )
{
  m_table[ reduce(v) ] = true;
}

void LookupTableIntOneMillion::remove( unsigned int v )
{
  m_table[ reduce(v) ] = false;
}

bool LookupTableIntOneMillion::contains( unsigned int v )
{
  return m_table[ reduce(v) ];
}
```

Implementation of New Member Function

```
unsigned int LookupTableIntOneMillion::reduce( unsigned int v
{
  return v % size;
}
```

Main

```
int main( void )
{
  LookupTableIntOneMillion table;

  table.add( 967 ); // Reduces to index 7
  table.add( 333 ); // Reduces to index 3
  table.add( 555 ); // Reduces to index 5

  return 0;
}
```

Space Complexity with a Reduction Function

- Ignoring conflicts, we now have space complexity linear with N
- Time complexity remains at O(1) access
- Achieving this space complexity cost us...
  - Increased work in order to double the size of the internal array
  - Increasing the work on each access
- Any other issues?

2.5. Mitigating the Weaknesses of Lookup Tables (cont.)

- Lookup tables are *inflexible* in storing non-integer types like strings.
- But strings, like any other type represented by a computer, are actually just numbers in the underlying representation!
- The key idea is to add a new function, a *hash* function, which converts a string or other arbitrary type into an integer.
  - Values of `char` type are just numbers!
  - The ASCII char 'a' is 97, 'b' is 98, etc...

Member function to convert a char array into an unsigned integer

```
unsigned int hash( char v[] ) {
```

Good Hash Functions

- What makes a hash function a “good” hash function?
- Property 1: We want a *valid* hash function
  - Returns the same value on subsequent calls to the same item
  - For any equivalent objects a == b, their hashes are also equal
- Property 2: We want a hash function that provides *uniformity*
  - Maps the expected inputs as evenly as possible over the output range
  - Specifically, the hash result should not be a value (e.g., 100) more often
- Property 3: We want a hash function with O(1) time complexity

Interface

```
class LookupTableStringHash
{
 public:
  LookupTableStringHash();
  void add      ( std::string v );
  void remove   ( std::string v );
  bool contains ( std::string v );

 private:
  unsigned int reduce ( unsigned int v );
  unsigned int hash   ( std::string  v );
  static const unsigned int size = 10;
  bool m_table[size];
};
```

Implementation

```
LookupTableStringHash::LookupTableStringHash()
{ }

void LookupTableStringHash::add( std::string v )
{
  m_table[ reduce( hash(v) ) ] = true;
}

void LookupTableStringHash::remove( std::string v )
{
  m_table[ reduce( hash(v) ) ] = false;
}

bool LookupTableStringHash::contains( std::string v )
{
  return m_table[ reduce( hash(v) ) ];
}
```

Implementation of New Member Functions

```
unsigned int LookupTableStringHash::reduce( unsigned int v )
{
  return v % size;
}

unsigned int LookupTableStringHash::hash( std::string v )
{
  unsigned int hashnum = 0;
  for ( auto& c : v )
    hashnum += c;
  return hashnum;
}
```

Main

```
int main( void )
{
  LookupTableStringHash table;

  table.add( "c" );   // Hash 99  : Reduced to 9
  table.add( "a" );   // Hash 97  : Reduced to 7
  table.add( "t" );   // Hash 116 : Reduced to 6

  table.add( "cat" ); // Hash: 99 + 97 + 116 = 312
                      // Reduced to 2

  return 0;
}
```

Summary

- We started off trying to mitigate the weaknesses of lookup tables

	`add`	`remove`	`contains`
list/vector	O(N)	O(N)	O(N)
tree	O(log(N))	O(log(N))	O(log(N))
lut	O(1)	O(1)	O(1)
lut w/ hash & reduce	O(1)	O(1)	O(1)

Strengths of Lookup Tables with `hash` and `reduce` Functions

- Time complexity is still O(1)

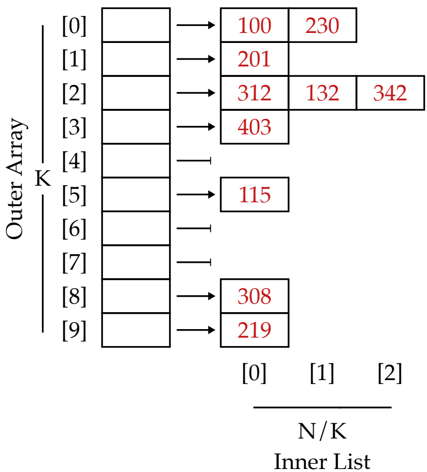
Mitigated Weaknesses of Lookup Tables with `hash` and `reduce`

- I want a set for a larger range of numbers (e.g., from 0 to 1,000,000)
  - Lookup tables were *space-inefficient*
  - Lookup tables with `hash` and `reduce` have space efficiency of O(N)
- I want a set of other types (e.g., strings)
  - Lookup tables were *inflexible*
  - Lookup tables with `hash` and `reduce` can handle any type as long as it provides a valid `hash` function
- What do we do about collisions?

3. Hash Tables

- Hash tables are lookup tables with `hash` and `reduce` functions that also handle collisions.
- Two common approaches for handling collisions
  - Separate chaining (usually with linked lists)
  - Open addressing (usually with linear probing)

3.1. Collision Handling with Separate Chaining



- The internal array elements are lists in a separate chaining scheme
- With a good hash function, the N elements are spread evenly over the K buckets.
- Adding a new element to the hash table is now a push back on a list
- What does this do to our time complexity?

```
class List
{
 public:

  class Itr
  {
   public:
    Itr( Node* node_p );
    void         next();
    std::string& get();
    bool         eq( const Itr& itr ) const;

   private:
    friend class List;
    Node* m_node_p;
  };

  Itr begin();
  Itr end();

  List();
  ~List();
  void push_front( std::string& v );

 private:

  struct Node
  {
    std::string value;
    Node*       next_p;
  };

  Node* m_head_p;
};
```

```
List::Itr    operator++( List::Itr& itr, std::string );
List::Itr&   operator++( List::Itr& itr );
std::string& operator* ( List::Itr& itr );
bool         operator!=( const List::Itr& itr0,
                         const List::Itr& itr1  );
```

- With the above List class (provided exactly as discussed in previous lectures), implement the `add` function of a hash table that handles collisions with chaining using linked lists.
- Note that a templated argument `Hasher` is often used to provide a class with a hash function.
  - The following class "has-a" `Hasher` named `hash`.
  - The `Hasher` has an overloaded call operator (i.e., `operator()`) that does the hash (e.g., calling `hash` with a string returns an unsigned integer).
- Below is the interface of the hash table. Assume that calling the `Hasher` object using `hash()` will create some collisions.

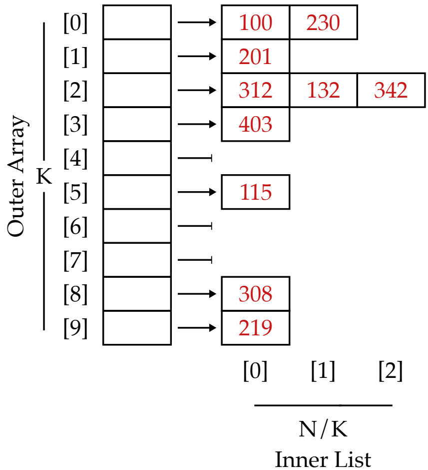
```
template < typename Hasher >
class HashTableChaining
{
 public:
  HashTableChaining();
  void add      ( const std::string& v );
  void remove   ( const std::string& v );
  bool contains ( const std::string& v );

 private:
  unsigned int reduce( unsigned int v );
  Hasher hash;
  static const unsigned int size = 10;
  List m_table[size];
};
```

```
void add( const std::string& v ) {
```

Summary

- How do hash tables compare to previous implementations of sets?



	`add`	`remove`	`contains`
list/vector	O(N)	O(N)	O(N)
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lut	O(1)	O(1)	O(1)
lut w/ hash & reduce	O(1)	O(1)	O(1)
hash table	________	________	________