

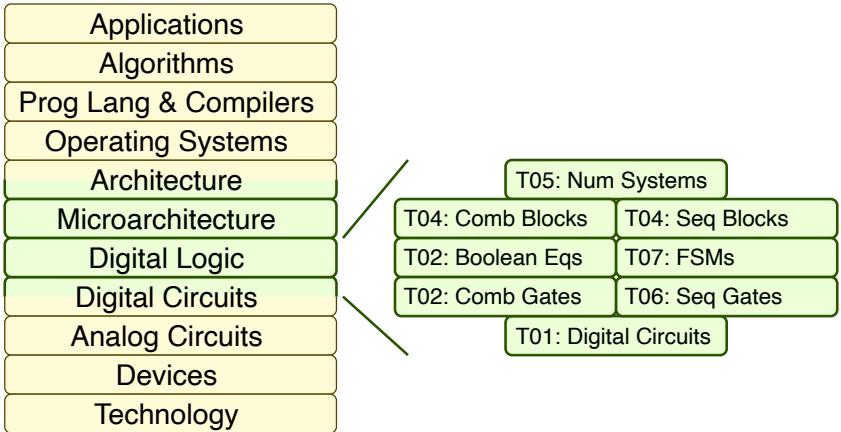
ECE 2300 Digital Logic and Computer Organization Fall 2024

Topic 5: Number Systems

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1. Unsigned Binary Number System

| $b_3b_2b_1b_0$ | v |
|----------------|-----|
| 0000 | |
| 0001 | |
| 0010 | |
| 0011 | |
| 0100 | |
| 0101 | |
| 0110 | |
| 0111 | |
| 1000 | |
| 1001 | |
| 1010 | |
| 1011 | |
| 1100 | |
| 1101 | |
| 1110 | |
| 1111 | |

$$v = b_3 \times 2^3 + b_2 \times 2^2 + b_1 \times 2^1 + b_0 \times 2^0$$

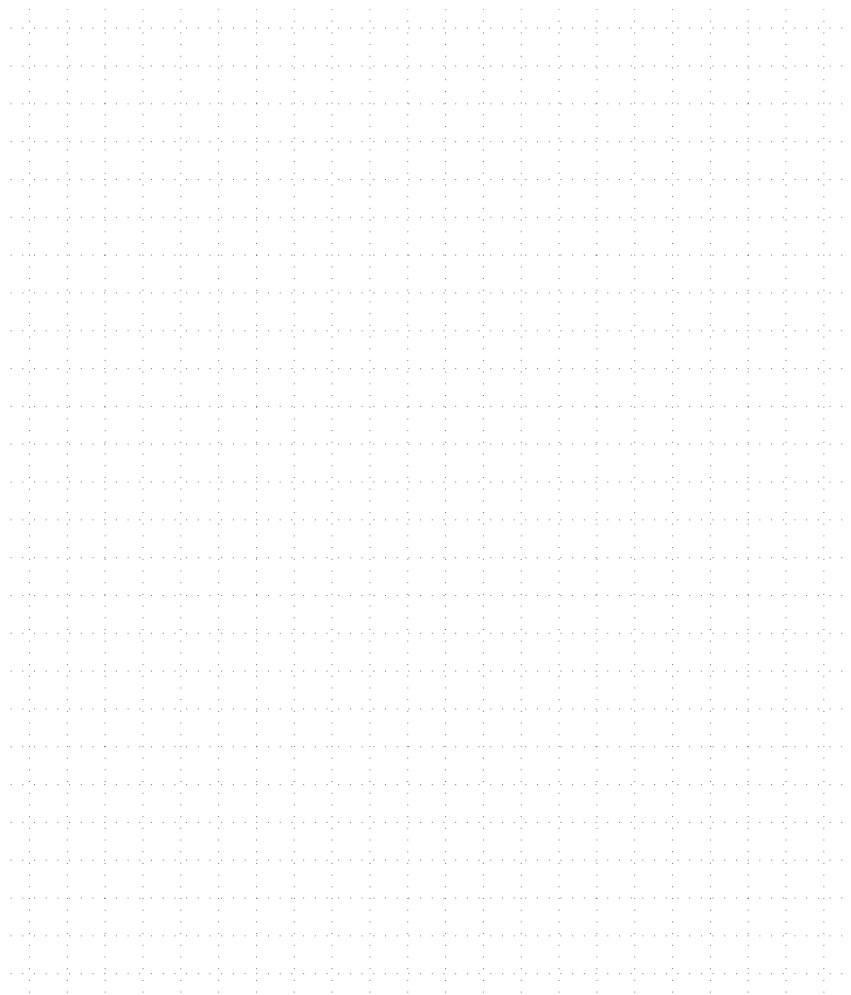
How do we represent $v = 13$?

What does $b = 1010$ represent?

How do we represent $v = 7$?

What does $b = 1110$ represent?

Implement an addsub unit for unsigned binary numbers



2. Sign Magnitude Number System

| $b_3b_2b_1b_0$ | v |
|----------------|-----|
| 0000 | |
| 0001 | |
| 0010 | |
| 0011 | |
| 0100 | |
| 0101 | |
| 0110 | |
| 0111 | |
| 1000 | |
| 1001 | |
| 1010 | |
| 1011 | |
| 1100 | |
| 1101 | |
| 1110 | |
| 1111 | |

$$v = -1^{b_3} \times (b_2 \times 2^2 + b_1 \times 2^1 + b_0 \times 2^0)$$

How do we represent $v = -5$?

What does $b = 1100$ represent?

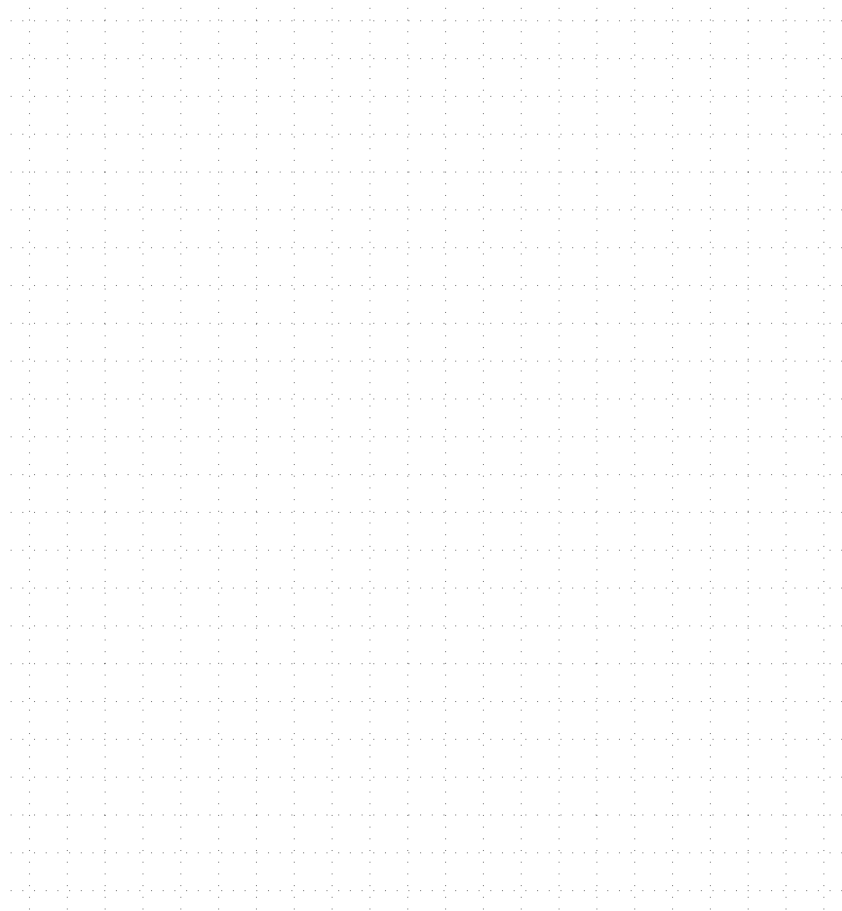
How do we represent $v = -7$?

What does $b = 0101$ represent?

Complementing a signed magnitude number

- Simply invert the sign bit (i.e., the most significant bit)

Implement an addsub unit for signed magnitude numbers



3. Two's Complement Number System

| $b_3b_2b_1b_0$ | v |
|----------------|-----|
| 0000 | |
| 0001 | |
| 0010 | |
| 0011 | |
| 0100 | |
| 0101 | |
| 0110 | |
| 0111 | |
| 1000 | |
| 1001 | |
| 1010 | |
| 1011 | |
| 1100 | |
| 1101 | |
| 1110 | |
| 1111 | |

$$v = b_3 \times -(2^3) + b_2 \times 2^2 + b_1 \times 2^1 + b_0 \times 2^0$$

How do we represent $v = -4$?

What does $b = 1110$ represent?

How do we represent $v = -1$?

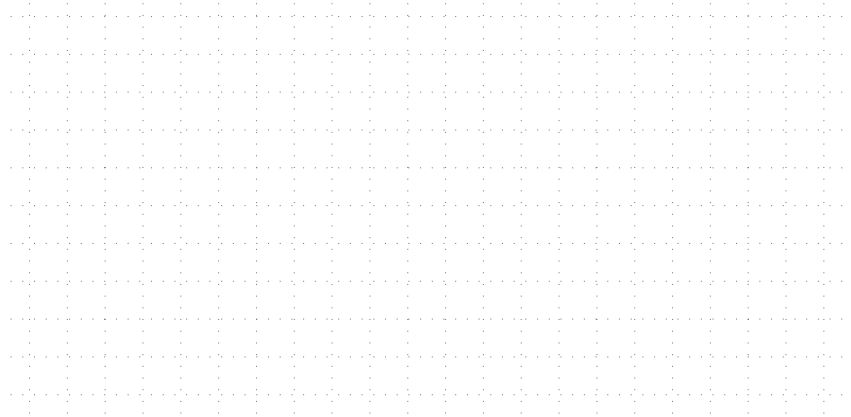
What does $b = 0101$ represent?

Complementing a two's complement number

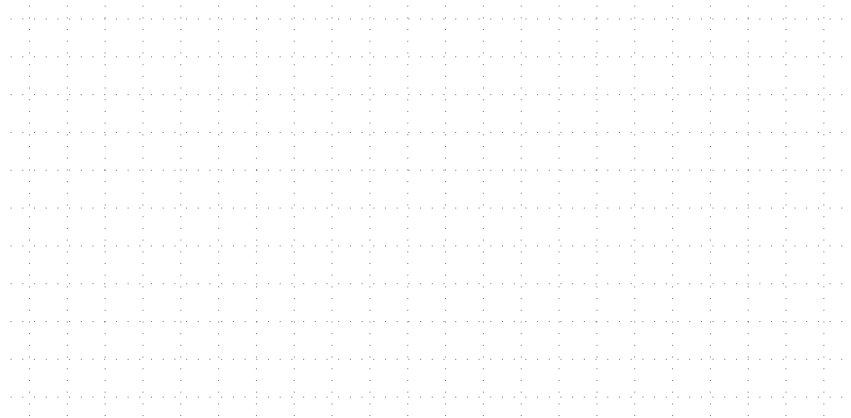
| v_{bin} | $b_3b_2b_1b_0$ | v_{2c} | $-v_{2c}$ | | | |
|-----------|----------------|----------|-----------|------|----|-------------|
| 0 | 0000 | 0 | 0 | 0000 | 0 | $16-0 = 16$ |
| 1 | 0001 | 1 | -1 | 1111 | 15 | $16-1 = 15$ |
| 2 | 0010 | 2 | -2 | 1110 | 14 | $16-2 = 14$ |
| 3 | 0011 | 3 | -3 | 1101 | 13 | $16-3 = 13$ |
| 4 | 0100 | 4 | -4 | 1100 | 12 | $16-4 = 12$ |
| 5 | 0101 | 5 | -5 | 1011 | 11 | $16-5 = 11$ |
| 6 | 0110 | 6 | -6 | 1010 | 10 | $16-6 = 10$ |
| 7 | 0111 | 7 | -7 | 1001 | 9 | $16-7 = 9$ |
| 8 | 1000 | -8 | -8 | 1000 | 8 | $16-8 = 8$ |
| 9 | 1001 | -7 | 7 | 0111 | 7 | $16-9 = 7$ |
| 10 | 1010 | -6 | 6 | 0110 | 6 | $16-10 = 6$ |
| 11 | 1011 | -5 | 5 | 0101 | 5 | $16-11 = 5$ |
| 12 | 1100 | -4 | 4 | 0100 | 4 | $16-12 = 4$ |
| 13 | 1101 | -3 | 3 | 0011 | 3 | $16-13 = 3$ |
| 14 | 1110 | -2 | 2 | 0010 | 2 | $16-14 = 2$ |
| 15 | 1111 | -1 | 1 | 0001 | 1 | $16-15 = 1$ |

3. Two's Complement Number System

- Two's complement of a 4-bit number v is $2^4 - v$
- Implementing this in hardware requires 5-bit subtractor (maybe?)



Implement an addsub unit for two's complement numbers



4. Fixed Point Number System

| $b_3b_2b_1b_0$ | v |
|----------------|-----|
| 0000 | |
| 0001 | |
| 0010 | |
| 0011 | |
| 0100 | |
| 0101 | |
| 0110 | |
| 0111 | |
| 1000 | |
| 1001 | |
| 1010 | |
| 1011 | |
| 1100 | |
| 1101 | |
| 1110 | |
| 1111 | |

$$v = b_3 \times 2^1 + b_2 \times 2^0 + b_1 \times 2^{-1} + b_0 \times 2^{-2}$$

How do we represent $v = 1.5$?

What does $b = 1011$ represent?

How do we represent $v = 3.25$?

How do we represent $v = 3.33$?

5. Floating Point Number System

| $b_3b_2b_1b_0$ | v |
|----------------|--------|
| 0000 | 0.2500 |
| 0001 | 0.3125 |
| 0010 | 0.3750 |
| 0011 | 0.4375 |
| 0100 | 0.5000 |
| 0101 | 0.6250 |
| 0110 | 0.7500 |
| 0111 | 0.8750 |
| 1000 | 1.0000 |
| 1001 | 1.2500 |
| 1010 | 1.5000 |
| 1011 | 1.7500 |
| 1100 | 2.0000 |
| 1101 | 2.5000 |
| 1110 | 3.0000 |
| 1111 | 3.5000 |

$$v = 2^{(b_3 \times 2^1 + b_2 \times 2^0) - 2} \times (1 + b_1 \times 2^{-1} + b_0 \times 2^{-2})$$

- b_3b_2 is called the exponent part
 - $b_3b_2 = 00$ represents 0.25 (subrange 0.25–0.50)
 - $b_3b_2 = 01$ represents 0.50 (subrange 0.50–1.00)
 - $b_3b_2 = 10$ represents 1.00 (subrange 1.00–2.00)
 - $b_3b_2 = 11$ represents 2.00 (subrange 2.00–4.00)
- b_1b_0 is called the fraction part
 - fraction part divides up subrange

How do we represent $v = 0.625$?



How do we represent $v = 0.33$?

