IMPLICANT

Product of multiple terms
\( \bar{A}B, \bar{A}C, D \)

MINTERM

Product involving all of the inputs to the function
\( \bar{A}ar{B}ar{C} \) is minterm for function of three variables
\( \bar{A}B \) is not a minterm for function of 3 variables

PRIME IMPLICANT

Cannot be combined with any other implicants
in equation to form a new implicant of fewer variables

\[
F = \bar{A}BC + \bar{A}ar{C} + \bar{A}ar{B}C
\]

\( \bar{A}BC \) not prime
\( \bar{A}ar{C} \) prime

\[
F = \bar{A}BC + \bar{A}ar{C}
\]

\( \bar{A}BC \) prime

\[
\begin{array}{c|ccc}
A \\
\hline
B & 00 & 01 & 11 & 10 \\
\hline
C & 0 & 1 & 1 & 0 \\
\end{array}
\]

\( \bar{A}ar{B} \)

ESSENTIAL PRIME IMPLICANT

Prime implicant that covers an output of function that
no other prime implicant is able to cover

COST

Assume cost is approximated by fewest implicants, "minimal cover"
After row dominance we are left with

\[
\begin{array}{ccc}
A'0 & A'D & A'C \\
2 & x & x \\
5 & x & x \\
7 & x & x \\
\end{array}
\]

Column A'D dominates 0'D

\[
\begin{array}{ccc}
A'0 & A'D & A'C \\
2 & x & x \\
5 & x & x \\
7 & x & x \\
\end{array}
\]

Row 7 dominates row 5

\[
\begin{array}{ccc}
A'0 & A'D & A'C \\
2 & x & x \\
5 & x & x \\
\end{array}
\]

A'D is an essential prime implicant

\[
\begin{array}{ccc}
A'0 & A'C \\
2 & x & x \\
\end{array}
\]

A'0 and A'C have common co-dominance so we can choose either one to cross out

So we are left with two optimal solutions

\[
F = A'D + A'D' \\
F = A'D + A'C
\]

Note that \( F = A'C + 0'D \) is also a minimal cover but it was eliminated through column dominance.

Takeaway: there can be many optimal solutions. CM is guaranteed to find a optimal solution but not all optimal solutions.
QM BRANCH-AND-BOUND ALGORITHM

1. GENERATE PRIME IMPLICANTS
2. CONSTRUCT PRIME IMPLICANT TABLE
3. REDUCE PRIME IMPLICANT TABLE
   3A. REMOVE ESSENTIAL PRIME IMPLICANTS
   3B. ROW DOMINANCE
   3C. COLUMN DOMINANCE
   3D. GOTO 3A UNTIL NO FURTHER REDUCTIONS POSSIBLE
4. SOLVE PRIME IMPLICANT TABLE
   4A. HEURISTICALLY CHOOSE A PRIME IMPLICANT
   4B. ASSUME CHOSEN PI IS IN MINIMAL COVER
      (CROSS OUT COLUMN + CORRESPONDING INTERSECTED ROWS)
   4C. GOTO 3
   4D. ASSUME CHOSEN PI IS NOT IN MINIMAL COVER
      (CROSS OUT COLUMN BUT NOT INTERSECTED ROWS)
   4E. GOTO 3
   4F. RETURN MINIMUM SOUTHS RETURNED FROM 4C AND 4E

IF AT ANY TIME SIZE OF CURRENT SOLUTION (IE NUMBER OF
PRODUCT TERMS) IS GREATER THAN BEST SOLUTION FOUND SO
FAR, RETURN FROM CURRENT RECURSIVE STEP IMMEDIATELY
<table>
<thead>
<tr>
<th>$A'$</th>
<th>$C$</th>
<th>$B$</th>
<th>$A'B$</th>
<th>$B'C$</th>
<th>$A'D$</th>
<th>$C'$</th>
<th>$AC'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
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<td></td>
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<tr>
<td>9</td>
<td></td>
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<tr>
<td>11</td>
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</tr>
</tbody>
</table>

Choose $A'C$, assume not in min cover.
Current solution: $\{A'D', A'C\}$

---

<table>
<thead>
<tr>
<th>$C$</th>
<th>$A'B$</th>
<th>$B'C$</th>
<th>$A'\overline{C}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

No essential prime implicants.
No row dominance.
Column dominance: $B'C > A'O$, $A'B > B'C$

Final solution: $\{A'D', A'C\}$

---

<table>
<thead>
<tr>
<th>$A'$</th>
<th>$C$</th>
<th>$B$</th>
<th>$A'B$</th>
<th>$B'C$</th>
<th>$A'D$</th>
<th>$C'$</th>
<th>$AC'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>9</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Choose $A'C$, assume not in min cover.
Current solution: $\{A'D'\}$

---

<table>
<thead>
<tr>
<th>$C$</th>
<th>$A'B$</th>
<th>$B'C$</th>
<th>$A'\overline{C}$</th>
</tr>
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<td>5</td>
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<td>X</td>
<td></td>
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<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Essential prime implicants: $B'C$, $A'O$
Current solution: $\{A'D', B'C, A'O\}$

---

<table>
<thead>
<tr>
<th>$C$</th>
<th>$A'B$</th>
<th>$B'C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

No row dominance.
Column dominance: $A' > B'C$, $A' > A'O$
Final solution: $\{A'D', B'C, A'O, A'C\}$
 Branch and Bound Search Tree

```
          [Root]
             /   \
            /     /
    A \ C   A \ C
     \    /   NOT IN COVER
      \  /     
       \ /      
      / \       
     /   \      
    /     \     
   /       \    
  /         \   
 /           \  
/             \ 
/               
/                 
/_________________
```

\[ A \bar{D}, A \bar{C}, \bar{C} \bar{E}, \bar{A} \bar{E} \]
\[ A \bar{D}, \bar{D} \bar{C}, \bar{C} \bar{E}, \bar{A} \bar{E} \]

Another Example

```
<table>
<thead>
<tr>
<th>A</th>
<th>O</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>X</td>
<td>X</td>
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<td>X</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

**Assume we have this implicit table after steps 1, 2, 3**

This is a cyclic cover!

Choose A is in M \cup \bar{C}

```
<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

Col Dom: E \geq B, E \geq C

Another cyclic cover!

Choose D is in solution

```
<table>
<thead>
<tr>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>
```

Choose J \notin solution

```
<table>
<thead>
<tr>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>3</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>
```

Sol = \{A, E, F\}

Now Dom 3 \geq 2, 3 \geq 4

Sol = \{A, D, E, F\} or \{A, D, F\}
**Choose A is not in the min cover**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>x</td>
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<td>2</td>
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<td>x</td>
<td>x</td>
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<td>3</td>
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<td></td>
<td>x</td>
<td>x</td>
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<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

**Col Dominance**

D > B, D > E, D > F

**SOL:** \{A, D\}

So best solution is \{A, D\} \cup 2 prime implicants

What if we started by choosing A is not in the min cover and then tried choosing A in the min cover?

Above would be the same as when we choose A in the min cover we would start similar to before:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>x</td>
<td>x</td>
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<td>4</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

**Branch and Bound search tree**

- A is in cover
  - A is not in cover
    - D is in cover
      - D φ  in cover
        - 2A, 3, F \{2A, 3, D,F\}
        - 2A, D, E \{2A, D, E\}
    - D φ  in cover
      - 2A, E, F \{2A, E, F\}
- By using a bound we can stop searching early while still guaranteeing optimality.