**BASIC CROSSBAR TOPOLOGY**

- Input terminals
- Output terminals
- Single stage, global crossbar difficult to scale in terms of cycle time, energy, and area
- Multi-stage topologies improve scalability, but raise many other interesting challenges

**Butterfly Topology**

- K-ary n-fly
- Router radix # stages
- Unidirectional channel
- 2-ary 2-fly
- Each router is similar to a 2x2 crossbar
\[ z - \text{Any} \]
\[ y - \text{FCLY} \]
\[ z^y = \text{nodes} \]
Constructing k-ary n-cube from k k-ary (n-1) cubes

2-ary 1-cube

2-ary 2-cube  
2-ary 3-cube

2-ary 4-cube

Hyper cubes

Binary 1-cube

Terminology

Cut

2-ary 2-fly

4-ary 2-cube

MESA
NODES + CHANNELS

- BIDIRECTIONAL CHANNELS IN MESH
- UNIDIRECTIONAL CHANNELS IN TFLY

CHANNELS

\[ W_C = \text{WIDTH (\# Wires)} \]
\[ f_C = \text{Frequency} \]
\[ t_C = \text{Latency} \]
\[ l_C = \text{Length} \]
\[ b_C = W_C \times f_C \quad \text{(Channel Bandwidth)} \]

- DIRECT (MESH) VS. INDIRECT (TFLY)
  (Note really too important a distinction)

Bisection Cuts

- Both CUT A and B on MESH are Bisection Cuts
- A is minimum bisection cut

\[ B_{BC} = \text{Bisection Channel Count (MESH)} \]
\[ B_{FUT} = 4 \]

\[ b_{BC} = B_{BC} \times b_C \quad \text{Bisection Bandwidth} \]
\[ B_{FUT} = 4 b_C \]

- Bisection BW is good way to estimate global wiring resources (ie. technology constraint)
PATHS

- Hop = one element or count on a path
- Hop count = number of hops on a path
- Hop count on path from terminal A → B on mesh shown earlier is
  \[ H_a = 5 \]
  \[ H_c = 4 \]
  \( H_a \) may or may not be included.
- Minimal path = smallest hop count between 2 terminals
- Diameter = \( (H_{\text{max}}) \) largest minimal path between all terminal pairs

Show on mesh + BFLY

\( H_{\text{max}} \) min = 8 \( H_{\text{max}} \) max = 4

- Average minimal hop count \( (H_{\text{avg}}) \) over all terminal pairs also is expected hop count for uniform arrivals

2D mesh \[ \frac{2}{3} k + 2 = \frac{2}{3} \cdot 4 + 2 = \frac{8}{3} + 2 = 4.3 \]
BFLY 4

PATH DIVERSITY

- mesh illustrate multiple ways to get from upper left to lower right terminal
- BFLY had no path diversity
- Adding an extra stage can get path diversity without impacting \( T_{\text{FL}} \) (section 3.6)
ADDING EXTRA
BFCY STAGES
↑ PAM DIVERSITY
Traffic Patterns

A traffic patterns include:

- Uniform
- Random
- Partition
- VS. Tape
- Neighbor
- General
- Transpose

For every entry, including sending to yourself.

Probability of src 3 sending a packet to dest 0.
In admission traffic patterns:

**Assume b/c**

\[ 0.256 + 0.256 = 0.51 \]

**going over 2 \rightarrow 3 channel**

Channel 2 \rightarrow 3 oversubscribed but so is existing channel to output terminal at Node 3

**Logical to physical mapping**

Assume partition traffic pattern:

Now are logical src/dst IDS in traffic pattern mapped to physical terminal IDs?

Mapping can turn any logical permutation pattern into any other of them.

Given permutation pattern usually assumes a specific mapping.
Performance: Throughput

Channel load ($N_c$) is amount of traffic that crosses channel $c$. If each input injects one unit of traffic according to given traffic pattern.

Example traffic pattern:
Source $i \rightarrow$ Dest $i+1$

Channel load ranges from 0 to $Z$.
Maximum channel load ($N_{MAX}$) is $Z$. These are the bottleneck channels that will limit throughput.

Alternative way to think about $N_c$.

Channel load is ratio of BUs demanded from channel ($c$) to input BUs injected by one terminal.
IDEAL THROUGHPUT:

\[ \Theta_{\text{term}} = \frac{b c}{N_{\text{max}}} \]

\[ \Theta_{\text{term}} = \frac{6}{\frac{1}{2}} = \frac{6}{2} = 3 \]

\[ \Theta_{\text{tot}} = N \cdot \frac{6}{N_{\text{max}}} = 8 \cdot \frac{6}{2} = 48 \]

Often interested in ideal throughput under UNI-LAND TRAFFIC.

RING EXAMPLE:

Two unidirectional channels.

Because symmetric, focus on this channel.

No minimal path 1 → 0.

Include if depends on routing algorithm.

Sometimes ideal throughput is for special routing algo.

Sometimes for ideal routing algo.

Use minimal routes for now.

So 3 minimal paths cross channel under consideration.

Each path has \( \frac{1}{3} \) units of traffic.

\[ N = 3 \cdot \frac{1}{3} = 1 \]

But this does not assume ideal routing. Ideal routing would load balance paths of length 2, so 0.5 traffic.

Count clockwise and 0.5 traffic count clockwise will

\[ N = \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = 0.5 \]
- Ideal routing will result in the lowest $\eta_{\text{max}}$ for given traffic pattern
- For any example and uniform random traffic

$\eta_{\text{max}} = 0.5$  $\Theta_{\text{term}} = \frac{6}{0.5} = 12$  $\Theta_{\text{tot}} = 4 \cdot \frac{6}{0.5} = 48$

**Small tricky examples**

Each tick = 1/4 unit

1. $\eta_{\text{max}} = 1$  $\Theta_{\text{term}} = 6$

2. $\eta_{\text{max}} = 1$  $\Theta_{\text{term}} = 6$

3. $\eta_{\text{max}} = 2$  $\Theta_{\text{term}} = 6 \frac{1}{2}$

4. $\eta_{\text{max}} = 4$  $\Theta_{\text{term}} = 6 \frac{1}{4}$
More generally for uniform random traffic

- On average with uniform random traffic, half the traffic crosses the direction.
- N total units of traffic, N/2 cross direction.
- Best routing will evenly balance load across direction channels (This was an issue in ring example).

\[ N_{\text{max}} = \frac{N/2}{Bc} = \frac{N}{2Bc} \]

\[ \Theta_{\text{rem}} = \frac{b}{N/2Bc} = \frac{2bBc}{N} = \frac{2Bc}{N} \]

\[ \Theta_{\text{tot}} = n \frac{2bBc}{N} = 2Bc \]

**Performance: Latency**

36 b/cycle = packet length \( L \)
8 b/cycle = b_c

Serialize packet into four units.

Packet 0
Packet 1

SSSSS
SSSSS
Pkt 1: Head Pkt -> S, Ro, R2, Lo, Li, Ro, Ri, El, D
Body Pkt -> S, Ro, R2, Lo, Li, Ro, Ri, El, D
Body Pkt -> S, Ro, R2, Lo, Li, Ro, Ri, El, D
Tail Pkt -> S, Ro, R2, Lo, Li, Ro, Ri, El, D

Serial Latency: \( L/6 \)
Channel Latency: \( t_c \)
Router Pipeline Latency: \( t_R \)

\[ T = T_{\text{Head}} + \frac{L}{6} \]

\[ T_{\phi} = H_R t_R + H_c t_c + \frac{L}{6} \]

Latency due to router hops
Latency due to channel hops
Serialization latency

Zero Load Latency (no contention)
Four Ways to Improve Latency

\[ T_\phi = H_r t_r + H_c t_c + \frac{L}{6} \]

- Shorter Routes
- Faster Routes
- Faster Channels

 Avg Latency vs Offered T\(3\)W

 New Ideal Routing + Flow Control

 Θ TOT
Activity

Assume:

L = 1286
b_c for mesh is 32 b/cycle
b_c for SMESH is 16 b/cycle — reduces channel bandwidth due to
T = 3
T_c = 1

Which topology can achieve higher ideal throughput under uniform random traffic?

(a) $\Theta_{term, mesh} > \Theta_{term, SMESH}$
(b) $\Theta_{term, mesh} < \Theta_{term, SMESH}$
(c) $\Theta_{term, mesh} = \Theta_{term, SMESH}$

Calculate zero load latency under uniform random traffic.
**MESU**

\[ T_c = 6 \quad T_b = 6 \cdot 32 = 192 \quad b/cycle \]

\[ \Theta_{tem} = \frac{2 \cdot T_b}{N} = \frac{2 \cdot 192}{6} = 64 \quad b/cycle \]

But \( b_c \) is just 32 \( b/cycle \)! Limited by water turbulence!

\[ \text{Achievable} \quad \Theta_{tem, \text{min}} = 32 \quad b/cycle \]

**SMESU**

\[ T_c = 10 \quad T_b = 10 \cdot 16 = 160 \quad b/cycle \]

\[ \Theta_{tem} = \frac{2 \cdot T_b}{N} = \frac{2 \cdot 160}{6} = 53 \quad b/cycle \]

But \( b_c \) is just 16 \( b/cycle \)! Again limited by water turbulence!

\[ \text{Achievable} \quad \Theta_{tem, \text{min}} = 16 \quad b/cycle \]
**MESY**

First calculate \( T_\phi \) under uniform random traffic

\[
T_\phi = W_{et_a} + W_{et_c} + \frac{L}{b} \\
= 2.59 \times 3 + 1.59 \times 1 + 128/32 \\
= 7.17 + 1.59 + 4
\]

\[T_\phi = 12.56 \text{ cycles}\]

**SMPY**

First calculate \( T_\phi \) under uniform random traffic

\[
T_\phi = W_{et_a} + W_{et_c} + \frac{L}{b} \\
= 2.06 \times 3 + 1.06 \times 1 + 128/16 \\
= 6.18 + 1.06 + 8
\]

\[T_\phi = 15.24 \text{ cycles}\]
SMESH

MESH

Traffic!

Acceptable θ term
(θ order)
Calculating $H_{M_{oo}}$

- Any 2-cube
- Circles = Terminals
- Squares = Routers

"Terminal Channels"

Calculating $H_{M_{oo_{2},i}}$

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<tr>
<th>src</th>
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1 x 4 = 4 including $i \rightarrow i$ 32/16 = 2
2 x 8 = 16 excluding $i \rightarrow i$ 28/12 = 2.3
3 x 4 = 12

Calculating $H_{M_{oo_{2},i}}$ with terminal channels

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2 x 4 = 8 including $i \rightarrow i$ 40/16 = 2.5
3 x 8 = 24 excluding $i \rightarrow i$ 40/12 = 3.3

4 x 4 = 16

Calculating $H_{M_{oo_{2},i}}$ without terminal channels

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1 x 8 = 8 including $i \rightarrow i$ 16/16 = 1
2 x 8 = 8 excluding $i \rightarrow i$ 16/12 = 1.3

Equation from book is $\frac{nk}{3}$ for even n

$\frac{nk}{3} = \frac{2 \times 2}{3} = 1.33$

This is for $H_{M_{oo_{2},i}}$

without terminal channels

+ ignoring i sending to itself