System-Level Implications of Residual Transmit-RF Impairments in MIMO Systems

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***Abstract*—Physical implementations of radio-frequency (RF) transceivers are known to suffer from a variety of non-idealities that defy proper compensation. As shown in Studer et al., WSA 2010, such *residual* transmit-RF (Tx-RF) impairments have a considerable impact on the performance of multiple-input multiple-output (MIMO) wireless communication systems, even for high-quality RF chains. In this paper, we analyze the tradeoffs arising from the dependencies of these residual Tx-RF impairments on system parameters, such as the antenna configuration and the transmit power. We find that careful selection of the transmit power (according to the given RF-chain implementation, antenna configuration, and link quality) enables to mitigate the performance loss caused by these distortions already at the transmitter.**

I. Introduction

Physical implementations of radio-frequency (RF) transmitters for wireless communication systems suffer from a variety of impairments that degrade the quality of the transmitted signal. The most prominent non-idealities at the transmit (Tx)-side are quantization error, carrier-frequency and sampling-rate offset, phase-noise, IQ-imbalance, and amplifier non-linearities [1]. While many of these impairments can be avoided through calibration or be compensated using sophisticated algorithms, a considerable amount of distortion defies proper compensation and therefore remains unaccounted for. In the remainder of the paper, we refer to these remaining distortions as *residual Tx-RF impairments*.

In [2] and [3] it was shown that residual Tx-RF impairments have a detrimental impact on the performance of real-world multiple-input multiple-output (MIMO) wireless communication systems, especially when operating in the high signal-to-noise ratio (SNR) regime. This performance degradation can partially be attributed to a mismatch between the real-world system that is affected by residual Tx-RF impairments and an idealistic system model which is used to develop (high-performance) MIMO detection algorithms. Luckily, this model mismatch can easily be compensated in the receiver [3]. However, the fundamental performance loss (in terms of channel capacity) associated with residual Tx-RF impairments *cannot* be mitigated by measures deployed at the receiver (e.g., using more sophisticated signal processing algorithms).

It is therefore natural to ask whether measures against residual Tx-RF impairments can be employed already at the *transmitter*. To answer this question, it is important to realize that increasing the Tx-power (with the goal of improving the link reliability) also increases the amount of distortions originating at the Tx-side. On the other hand, lowering the Tx-power level reduces these distortions but also degrades the link reliability due to thermal noise in the receiver. Apparently, there is a system-level tradeoff between the amount of residual Tx-RF impairments and the signal to thermal-noise ratio at the receiver, which can be adjusted to maximize performance.

1) Contributions: In this paper, we consider the following facts: i) A significant portion of the residual Tx-RF impairments can be attributed to non-linear distortions in the power amplifier and ii) the amount of distortion increases with the Tx-power. We study the impact of this dependency in non-ideal (real-world) Tx-RF chains on the system performance with the goal to maximize the ergodic channel capacity. In particular, we study the following aspects:

- We analyze the channel capacity of a MIMO system model that includes residual Tx-RF impairments and we point out the existence of two different operating regimes.
- We investigate the impact of increasing the number of Tx-antennas on channel capacity under the usual regulatory constraints that enforce a maximum equivalent isotropic radiated power (EIRP).
- We discuss the impact of the Tx-power level on the system capacity, taking into account the limitations of real-world Tx-RF chains

2) Notation: Matrices are set in boldface capital letters and vectors in boldface lower-case letters. $\mathbf{A}^H$ and $\mathbf{A}^{-1}$ stands for the conjugate transpose and the inverse of the matrix $\mathbf{A}$, respectively. The $N \times N$ identity matrix is denoted by $\mathbf{I}_N$. The Euclidean norm of the vector $\mathbf{x}$ is $\|\mathbf{x}\|$. Sets are designated by uppercase calligraphic letters and $\mathbb{E}[\cdot]$ refers to the expectation operator.

3) Outline: The remainder of the paper is organized as follows. Sec. II reviews relevant previous results on residual Tx-RF impairments in MIMO systems and characterizes the corresponding channel capacity. In Sec. III system-level implications of residual Tx-RF impairments are investigated and the associated tradeoffs are studied. We conclude in Sec. IV.

II. System Model for Residual Tx-RF Impairments

We briefly review the system model developed in [3] for residual Tx-RF impairments. We then describe the considered performance measures and finally introduce a model for the relationship between Tx-power and the amount of distortions in real-world Tx-RF chains.

A. MIMO System Model with Tx-Noise

We consider a MIMO wireless communication system with M_{T} transmit and $M_{\mathrm{R}} \geq M_{\mathrm{T}}$ receive antennas. With spatial multiplexing, the information bits are mapped to M_{T}-dimensional transmit vectors $\mathbf{s} \in \mathcal{O}^{M_{\mathrm{T}}}$, where $\mathcal{O}$ denotes the set of complex-valued constellation points. The Tx-power per antenna can be adjusted to σ_{s}^2 by scaling the entries of $\mathcal{O}$ such that $\mathbb{E}\left[\mathbf{s}\mathbf{s}^H\right] = \sigma_{\mathrm{s}}^2 \mathbf{I}_{M_{\mathrm{T}}}$. In order to account for residual Tx-RF impairments, we consider the (impaired) baseband input-output relation [3]

$$\mathbf{y} = \gamma \mathbf{H}(\mathbf{s} + \mathbf{n}_{\mathrm{t}}) + \mathbf{n}_{\mathrm{r}} \tag{1}$$

where $\mathbf{y}$ is the M_{R}-dimensional receive-vector and $\mathbf{H}$ denotes the $M_{\mathrm{R}} \times M_{\mathrm{T}}$ complex-valued channel matrix having i.i.d. circularly-symmetric complex Gaussian (CSCG) distributed entries with unit variance. The residual Tx-RF impairments are modeled by the *Tx-noise* vector $\mathbf{n}_{\mathrm{t}}$, which is assumed to be i.i.d. CSCG distributed with variance σ_{t}^2 per complex dimension. Note that this assumption was shown to accurately reflect the behavior of real-world residual Tx-RF impairments in MIMO systems employing orthogonal frequency-division multiplexing (OFDM) [3]. The real-valued parameter $\gamma \in (0, 1]$ reflects the signal attenuation (i.e., path loss) between the transmitter and the receiver. The thermal noise at the receiver $\mathbf{n}_{\mathrm{r}}$ is assumed to be i.i.d. CSCG with variance σ_{r}^2 per complex dimension.

For the model in (1), the average signal-to-noise ratio (SNR) per receive antenna corresponds to

$$\mathsf{SNR} = \gamma^2 \sigma_{\mathrm{s}}^2 M_{\mathrm{T}} / \sigma_{\mathrm{r}}^2.$$

Since the total Tx-power $M_{\mathrm{T}}\sigma_{\mathrm{t}}^2$ and thermal noise power σ_{r}^2 usually remain constant in practical systems, the SNR is solely determined by the signal attenuation γ^2. In the remainder of the paper, we assume perfect channel state information (CSI) at the receiver. The transmitter is only assumed to know the variances σ_{r}^2, σ_{t}^2, the signal attenuation γ^2, and the statistics of the channel matrix $\mathbf{H}$.

B. Channel Capacity

In order to investigate the fundamental impact of Tx-noise on the performance of a MIMO communication systems, we consider the channel capacity of (1) which corresponds to [3]

$$C(\mathbf{H}) = \log_2 \det\left(\mathbf{I}_{M_{\mathrm{R}}} + \sigma_{\mathrm{s}}^2 \gamma^2 \mathbf{K}^{-1} \mathbf{H}\mathbf{H}^H\right) \tag{2}$$

bits per channel use (in bit/s/Hz), where

$$\mathbf{K} = \sigma_{\mathrm{t}}^2 \gamma^2 \mathbf{H}\mathbf{H}^H + \sigma_{\mathrm{r}}^2 \mathbf{I}_{M_{\mathrm{R}}}$$

is the covariance matrix that includes an i.i.d. contribution from the thermal noise and a colored contribution $\sigma_{\mathrm{t}}^2\gamma^2\mathbf{H}\mathbf{H}^H$, which is caused by the Tx-noise model. Equivalently, the channel capacity (2) can be written as

$$C(\mathbf{H}) = \sum_{i=1}^{M_{\mathrm{T}}} C_i \;\text{ with }\; C_i = \log_2\left(1 + \frac{\sigma_{\mathrm{s}}^2 \lambda_i}{\lambda_i \sigma_{\mathrm{t}}^2 + \sigma_{\mathrm{r}}^2/\gamma^2}\right) \tag{3}$$

where, λ_i, $i = 1, \ldots, M_{\mathrm{T}}$ correspond to the eigenvalues (EVs) of the matrix $\mathbf{H}\mathbf{H}^H$.

C. Operating Regimes

It is important to realize that the channel capacity expression in (3) gives rise to two different operation regimes:

1) Thermal-noise-limited regime: If the transmitted signal is severely attenuated, which naturally occurs for *long links* (i.e., large distances between the transmitter and the receiver), the thermal noise dominates in the capacity expression (3). More precisely, if $\sigma_{\mathrm{r}}^2/\gamma^2 \gg \lambda_i\sigma_{\mathrm{t}}^2, \forall i$, (3) approaches the (well-known) channel capacity without Tx-noise [4]

$$C(\mathbf{H}) \approx \sum_{i=1}^{M_{\mathrm{T}}} \log_2\left(1 + \frac{\lambda_i \sigma_{\mathrm{s}}^2}{\sigma_{\mathrm{r}}^2/\gamma^2}\right). \tag{4}$$

2) Tx-noise-limited regime: On the other hand, if the signal is only weakly attenuated, which is the case for *short links* (i.e., small distances between the transmitter and the receiver), the channel capacity is dominated by the Tx-noise. In this region, we have $\sigma_{\mathrm{r}}^2/\gamma^2 \ll \lambda_i\sigma_{\mathrm{t}}^2, \forall i$, and consequently, the channel capacity (3) approaches the upper bound [3]

$$C(\mathbf{H}) \leq M_{\mathrm{T}} \log_2\left(1 + \frac{\sigma_{\mathrm{s}}^2}{\sigma_{\mathrm{t}}^2}\right). \tag{5}$$

Note that in practical systems the thermal-noise level is significantly smaller than the transmit-noise level (i.e., $\sigma_{\mathrm{r}}^2 \ll \sigma_{\mathrm{t}}^2$), which means that "weak attenuation" can still correspond to levels between $-80\,\mathrm{dB}$ and $-90\,\mathrm{dB}$.

D. EVM of Real-World Tx-RF Chains

In the RF-literature, the residual Tx-RF impairments are routinely expressed in terms of the error-vector magnitude (EVM), which corresponds to a lump-sum measure that characterizes the quality of the transmitted signals. Formally, the EVM is defined as [5]

$$\mathsf{EVM} = \frac{\mathbb{E}\left[\|\tilde{\mathbf{s}} - \mathbf{s}\|^2\right]}{\mathbb{E}\left[\|\mathbf{s}\|^2\right]}$$

where $\mathbf{s}$ and $\tilde{\mathbf{s}}$ are the baseband equivalent of the desired (ideal) signal and the actual output signal of the RF transmitter, respectively.[1] For the Tx-noise model introduced in Sec. II-A, we have $\tilde{\mathbf{s}} = \mathbf{s} + \mathbf{n}_{\mathrm{t}}$ and the EVM is given by $\mathsf{EVM} = \sigma_{\mathrm{t}}^2/\sigma_{\mathrm{s}}^2$.

A review of state-of-the-art integrated Tx-RF circuits (see, e.g., [6]–[11] and references therein) shows that the EVM typically exhibits a strong dependency on the Tx-power σ_{s}^2. As an example, the EVM characteristics of an integrated Tx-RF transceiver (including the power amplifier) reported in [6]

[1] In practical systems, the EVM is typically measured *after* calibration of the RF-chain and *after* compensation of those impairments for which corresponding practical compensation algorithms exist.

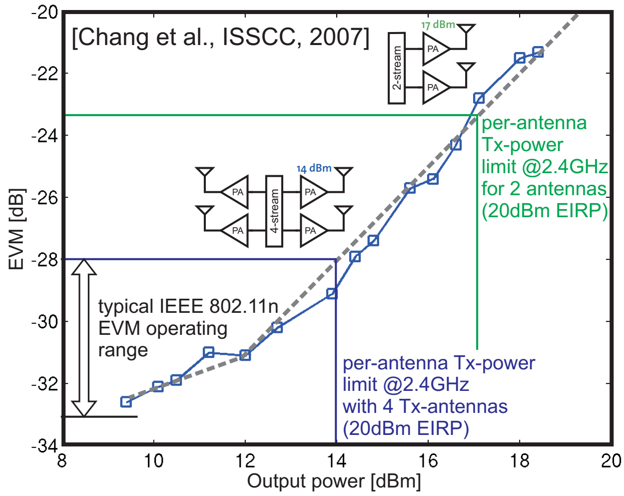

Fig. 1. EVM versus output power σ_s^2 for a typical integrated IEEE 802.11g 2.4/5.2 GHz RF chain including an on-chip power amplifier [6].

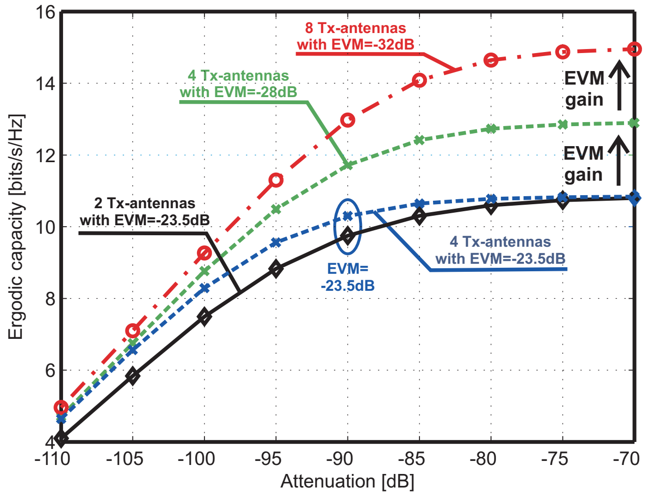

Fig. 2. Impact of increasing the number of Tx-antennas on ergodic MIMO channel capacity with two receive antennas under a constant power constraint.

is shown in Fig. 1. On a double logarithmic scale, we observe that in the range of interest (i.e., between 12 dBm and 19 dBm), the EVM increases roughly linearly as the Tx-power increases. A similar behavior can also be observed for other RF-chain implementations. Hence, we assume the following (linear) model for the subsequent analysis:

$$\log_{10}(\mathsf{EVM}) = \log_{10}\left(\frac{\sigma_t^2}{\sigma_s^2}\right) = a\log_{10}(\sigma_s^2) + b. \qquad (6)$$

For all numerical simulations shown in the remainder of the paper, the paramters a and b in (6) are chosen according to the EVM-characteristics of the Tx-RF-chain implementation reported in [6] (cf. Fig. 1).

III. System-Level Implications

In order to analyze the impact of residual Tx-RF impairments on the performance of MIMO systems, we rely on the *ergodic* channel capacity $C_e = \mathbb{E}[C(\mathbf{H})]$ (averaged over channel realizations) as an upper limit on the achievable transmission rate of practical systems.

A. Impact of Number of Tx-Antennas

In practical wireless communication systems, regulations and standards limit the maximum EIRP. For MIMO systems without CSI at the transmitter (i.e., without knowledge of the instantaneous channel realization $\mathbf{H}$), this available power budget P_{tot} is typically distributed equally across all transmit antennas so that $\sigma_s^2 = P_{\mathrm{tot}}/M_{\mathrm{T}}$.

In order to investigate the impact of different antenna configurations, we start from a baseline 2×2 (referring to $M_{\mathrm{T}}\times M_{\mathrm{R}}$) system with $P_{\mathrm{tot}} = 20$ dBm, which is the EIRP limit for the 2.4 GHz ISM band in Europe.[2]

In that case, each of the two Tx-RF chains needs to provide an output power of 17 dBm which, according to Fig. 1, results in an EVM of -23.5 dB.

[2]We compute the thermal-noise variance σ_r^2 by assuming 40 MHz bandwidth and a temperature of 293 K.

We then increase the number of Tx-antennas and compare the ergodic capacity C_e with that of a 4×2, and a 8×2 MIMO system. From Fig. 2 we observe that merely increasing the number of Tx-antennas to $M_{\mathrm{T}} = 4$, while still assuming an EVM of -23.5 dB, yields only a negligible capacity advantage for weak attenuations (i.e., for short links). However, when taking into account that the reduced per-antenna output power also results in an improved EVM value (cf. Fig. 1), we immediately observe a considerable increase of 2 bits/s/Hz in terms of ergodic capacity for the 4×2 and also for the 8×2 configuration. Hence, increasing the number of Tx-antennas and distributing the available power budget P_{tot} equally across all Tx-antennas leads to a significant reduction of the residual Tx-RF impairments already at the transmitter and eventually increases the system capacity. We refer to this capacity increase as *EVM gain*.

B. Selection of the Optimum Tx-Power Level

As described in Sec. II-C, for MIMO systems impaired by Tx-noise, we can identify two operating regimes: For long links, the channel capacity is mostly determined by the thermal noise, while the Tx-noise dominates for short links. At the same time, we have observed from Fig. 1 that increasing the total Tx-power $P_{\mathrm{tot}} = M_{\mathrm{T}}\sigma_s^2$ improves the SNR at the expense of a degradation in terms of EVM. Considering the dominant terms in both operating regimes, it is evident that for long links (cf. (4)) it is advantageous to improve the SNR by increasing the output power even at the expense of more residual distortions. Conversely, for short links (cf. (5)), the Tx-noise should be reduced by lowering the Tx-power level even at the expense of degrading the SNR.

To illustrate this observation, Fig. 3 shows the ergodic capacity as a function of the signal attenuation (distance) for different Tx-power levels in a 4×4 MIMO system. The figure clearly reveals both operating regimes and demonstrates the advantage of reducing the Tx-power for weak levels of signal

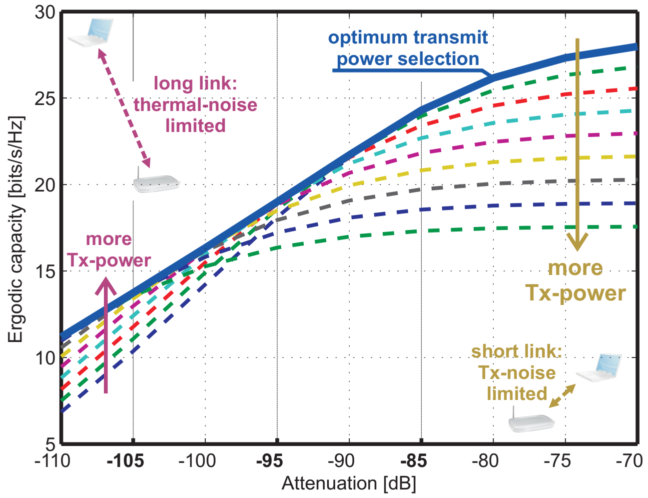

Fig. 3. Impact of increasing the Tx-power on ergodic capacity as a function of the channel attenuation in a 4×4 MIMO system.

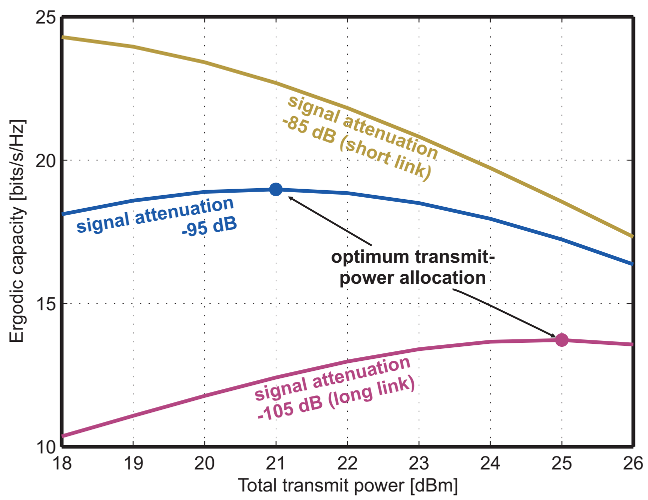

Fig. 4. Impact of increasing the Tx-power on ergodic capacity for -85 dB, -95 dB, and -105 dB attenuation in a 4×4 MIMO system.

attenuation (i.e., for short links). Fig. 4 details the impact of the output power on the ergodic capacity for three levels of signal attenuation. We observe that for each attenuation level an *optimum Tx-power level* (in terms of ergodic capacity) exists, which balances the advantage of better SNR with the associated increase of the EVM. Choosing this optimum Tx-power level for each attenuation, eventually results in the capacity enveloppe shown in Fig. 3.

We finally note that we select the Tx-power level based on the signal attenuation γ^2 and not on the individual channel realization $\mathbf{H}$. Hence, only knowledge of γ^2 is required at the transmitter, which can easily be acquired using CSI feedback mechanisms or by exploiting the reciprocity of wireless channels. Our own simulations have shown that an instantaneous adjustment of the Tx-power level (in accordance to the realization of $\mathbf{H}$) provides only a marginal advantage in terms of ergodic capacity for the considered system configuration.

IV. Conclusions

MIMO wireless communication systems suffer not only from thermal noise but also from residual transmit (Tx)-RF impairments. In real-world systems, the amount of these distortions exhibits a strong dependency on the Tx-power used per RF chain.

In this paper, we have shown that considering the dependency of the signal quality on the transmitted power level bares the potential for improving the ergodic capacity of MIMO wireless communication systems. In particular, we demonstrated that choosing the Tx-power level in accordance with the employed RF-implementation, the antenna configuration, and the channel attenuation, can significantly reduce residual Tx-RF impairments already at the transmitter, eventually leading to significant improvements in terms of ergodic capacity.

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